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**IMPACT OF FINITE FREQUENCY DEVIATION ON THE PERFORMANCE  
OF DUAL-FILTER HETERODYNE FSK LIGHTWAVE SYSTEMS**

1991

Ozan K. Tonguz, *Member, IEEE*, and M. Okan Tanrikulu, *Student Member, IEEE*

Photonics Research Center

Department of Electrical and Computer Engineering

State University of New York at Buffalo

Buffalo, NY 14260

USA

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Leonid G. Kazovsky<sup>1</sup>, *Fellow, IEEE*

Department of Electrical Engineering

Stanford University

Stanford, CA 94305-4055

USA

**92-05237**



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<sup>1</sup>The work of Leonid Kazovsky was partially supported by ONR under grant number N00014-91-J-1857.

## Abstract

A detailed theoretical analysis is given for the impact of finite frequency deviation on the sensitivity of dual-filter heterodyne Frequency Shift Keying (FSK) lightwave systems. Our analysis provides closed-form signal-to-noise ratio (SNR) results for estimating the bit-error-ratio (BER) performance of the system. These closed-form results provide an insight into the impact of finite frequency deviation  $2\Delta f_d$ , laser linewidth  $\Delta\nu$ , bit rate  $R_b$ , and IF filter bandwidths on the system performance. It is shown that there is a well-defined relationship between the choice of frequency deviation and the tolerable amount of laser phase noise. When there is no phase noise, a frequency deviation of  $2\Delta f_d = 0.72R_b$  is sufficient for 1 dB sensitivity penalty with respect to infinite frequency deviation case; whereas for a linewidth of  $\Delta\nu = 0.50R_b$  the required frequency deviation increases to  $2\Delta f_d = 3.42R_b$  for the same sensitivity penalty. The sensitivity degradation can be very severe for a fixed linewidth as the frequency deviation gets smaller: for a linewidth of 20% the sensitivity penalty is only 0.54 dB when the frequency deviation is infinite whereas it is 3.48 dB when the frequency deviation is  $2\Delta f_d = R_b$ . We also quantify the impact of finite frequency deviation on optimum IF filter bandwidths. For a fixed linewidth, the optimum IF filter bandwidth decreases as frequency deviation becomes smaller: for  $\Delta\nu = 0.5R_b$  the optimum IF filter bandwidth reduces from  $7R_b$  to  $3R_b$  when  $2\Delta f_d$  reduces from very large values to  $3R_b$ .



Statement A per telecon  
Dr. Rabinder Madan ONR/Code 1114  
Arlington, VA 22217-5000

NWW 3/16/92

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## I. INTRODUCTION

The FSK heterodyne dual filter and single filter detection systems are examples of asynchronous lightwave systems, whereby a relatively large laser diode (LD) spectral spread can be tolerated. That makes the use of conventional DFB LD's possible which in turn is important for achieving a simple and stable system [1]-[8]. The dual filter detection system is particularly attractive because it offers a 3-dB higher receiver sensitivity than the single filter detection system [1]. The ideal performance of dual filter heterodyne FSK lightwave systems were previously studied by several authors assuming ideal conditions for the intermediate frequency (IF) and the frequency deviation between the two frequencies of data transmission [2], [9], [10].

In particular, in [2], [9] and [10] the sensitivity degradation due to laser phase noise and shot noise was studied assuming that the IF and the frequency separation between the two tones are infinitely large. In practical systems, however, these assumptions are not valid [1], [3], [4], [11], [12]. In this paper, we study the impact of finite frequency deviation on the system performance. Our analysis also includes the impact of laser phase noise and additive shot noise.

We show that there is a well-defined relationship between the choice of frequency deviation and the tolerable amount of laser phase noise for a prescribed level of sensitivity degradation (e.g., 1 dB). Our results indicate that when there is no phase noise, a frequency deviation  $2\Delta f_d = 0.72R_b$  is sufficient for 1 dB sensitivity penalty with respect to infinite frequency deviation case; whereas, for an IF linewidth of  $\Delta\nu = 0.50R_b$  the required frequency deviation increases to  $2\Delta f_d = 3.42R_b$  for the same sensitivity penalty. The influence of finite frequency deviation on the values of other important system parameters such as optimum IF filter bandwidth and bit-error-ratio (BER) is also quantified. It is demonstrated that for a fixed IF linewidth value, the optimum IF filter bandwidth decreases as the frequency separation between the two tones becomes smaller. As an example, for  $\Delta\nu = 0.5R_b$  the optimum IF filter bandwidth required reduces from  $7R_b$  to  $3R_b$  when  $2\Delta f_d$  reduces from very large

values to  $3R_b$ . Using the BER results computed, sensitivity penalty due to finite frequency deviation is quantified as a function of IF laser linewidth. Our results show that the sensitivity degradation can be very severe for a fixed linewidth as the frequency deviation gets smaller. As an example, for an IF linewidth of 20% the sensitivity penalty is only 0.54 dB when the frequency deviation is extremely large (infinite); whereas it is 3.48 dB when the frequency deviation is  $2\Delta f_d = R_b$ . In this paper, we also investigate practically important suboptimum cases which do not use optimum IF filter bandwidths. More specifically, we quantify the sensitivity degradation due to finite frequency deviation for a fixed linewidth value when the IF filter bandwidth used is not optimized for different frequency deviations.

The rest of this paper is organized as follows. In Section II we give a system description and problem statement. Section III contains basic receiver equations, the signal-to-noise ratio, the bit-error rate, and the main results of the paper. A physical interpretation of the main results obtained is presented in Section IV. Finally, Section V contains the conclusions of this study.

## II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

The block diagram of the dual filter optical heterodyne FSK receiver is shown in Figure 1. It was shown in [9] that the total output current processed by such a receiver is

$$i_T(t) \equiv \begin{cases} A_S \cos[(\omega + \Delta\omega)t + \phi(t)] + n(t), & \text{for data} = 1 \\ A_S \cos[(\omega - \Delta\omega)t + \phi(t)] + n(t), & \text{for data} = 0 \end{cases} \quad (1)$$

where  $A_S$  is the signal amplitude;  $n(t)$  is the total noise process at the output of the balanced receiver;  $\omega = 2\pi f_{IF}$  is the intermediate frequency (IF) in radians per second;  $\phi(t)$  is the total phase noise due to the transmitter and local oscillator (LO) lasers; and  $\Delta\omega = 2\pi\Delta f_d$  is the frequency deviation in radians/sec. These quantities are given by the following expressions:

$$A_S \equiv 2R\sqrt{P_S P_{LO}} \quad \text{amperes} \quad (2)$$

$$n(t) \equiv n_1(t) - n_2(t) \quad \text{amperes} \quad (3)$$

$$\omega \equiv \omega_S - \omega_{LO} \quad \text{radians/second} \quad (4)$$

$$\phi(t) \equiv \phi_S(t) - \phi_{LO}(t) \quad \text{radians} \quad (5)$$

$$2\Delta\omega \equiv \omega_1 - \omega_0 \quad \text{radians/second} \quad (6)$$

where  $R$  is the detector responsivity;  $n_1(t)$  and  $n_2(t)$  are the noises of the two photodiodes;  $\omega_S$  and  $\omega_{LO}$  are the frequencies of the signal and the LO, respectively;  $\phi_S(t)$  and  $\phi_{LO}(t)$  are the phase noises of the transmitter and LO lasers, respectively; and  $\omega_1$  and  $\omega_0$  are the frequencies of transmission for  $data = 1$  and  $data = 0$ , respectively.

The total phase noise  $\phi(t)$  is defined by expression (5). Its derivative  $\dot{\phi}(t)$  has a zero-mean Gaussian probability density function (pdf); the single-sided power spectral density (PSD) of  $\dot{\phi}(t)$  is given by

$$S_{\dot{\phi}(t)}(f) = 4\pi\Delta\nu \quad 0 < f < \infty \quad (7)$$

where  $\Delta\nu$  is the FWHM linewidth at the IF, i.e.,

$$\Delta\nu = \Delta\nu_T + \Delta\nu_{LO} \quad (8)$$

where  $\Delta\nu_T$  and  $\Delta\nu_{LO}$  are the linewidths of the transmitter and LO lasers, respectively. The PSD shape in (7) implies the Lorentzian laser lineshape.

The additive noise  $n(t)$  is composed of both the shot noise and the thermal noise. The single-sided PSD of  $n(t)$  is

$$S_n \equiv \eta \quad \text{for } 0 < f < \infty \quad \text{amperes}^2 \text{ per hertz} \quad (9)$$

where

$$\eta \equiv 2eRP_{LO} + \eta_{TH} \text{ amperes}^2 \text{ per hertz.} \quad (10)$$

In (10),  $e$  is the electron charge and  $\eta_{TH}$  is the PSD of the thermal noise. Expression (9) implies that the additive noise is white; this assumption may be inaccurate in systems having wide laser linewidth or large bit rate. The autocorrelation function of  $n(t)$  is given by

$$R_n(t_1, t_2) \equiv E[n(t_1)n(t_2)] = 0.5\eta\delta(t_1 - t_2) \quad (11)$$

In previous studies on dual filter heterodyne FSK lightwave systems, the frequency separation between the two FSK frequencies  $2\Delta\omega$  was assumed to be sufficiently large, so that the crosstalk between the two filters was negligible [2], [9], [10]. For practical system design, however, such an assumption is not valid. Specifically, in multichannel lightwave systems employing dual-filter heterodyne FSK receivers, the frequency separation between the two tones is a significant system design parameter which influences the number of channels that can be received for a prescribed level of crosstalk [13]-[17].

The physical scenario under consideration is shown in Figure 2. Figure 2(a) illustrates the case when the two frequencies are sufficiently apart from each other so that there is negligible crosstalk. The information-bearing signal may also contain phase noise. As the frequency deviation becomes smaller the two information-bearing signals start overlapping (see Fig. 2(b)) and the performance of the system under investigation deteriorates. One of the objectives of this study is to quantify the required frequency deviation for a given linewidth and a fixed sensitivity penalty (e.g., 1 dB). Another important case is shown in Figure 2(c). For a fixed frequency deviation, assuming the two information-bearing signals have no phase noise, there is a certain performance level (and a corresponding BER) which depends on the frequency spacing between  $\omega_1$  and  $\omega_0$ . Clearly, if the

frequency deviation is kept fixed, as the two signals are impaired by more and more phase noise, the crosstalk due to laser phase noise becomes more severe; and the system performance deteriorates sharply. This physical situation is illustrated in Fig. 2(d). Another objective of this paper is to quantify the laser linewidth which can be tolerated for a prescribed level of sensitivity penalty, such as 1 dB, and for a fixed value of frequency deviation.

The physical scenario described above can only be analyzed if certain other system parameters are also carefully engineered. Note in Figures 2(a) and 2(b), for example, that as the frequency deviation gets narrower the optimum IF filter bandwidth required becomes smaller. Thus, to optimize the system performance it is essential to evaluate the optimum IF filter bandwidth corresponding to a fixed (but finite) frequency deviation and a fixed linewidth in the physical situation depicted by Figures 2(a) and 2(b). Computation of optimum IF filter bandwidths is also a necessity for the optimum performance evaluation of the system described by Figures 2(c) and 2(d). Obviously, one can choose not to optimize the IF filter bandwidth; in that case, however, a certain sensitivity penalty must be paid and the system performance is no longer optimum.

### III. SYSTEM PERFORMANCE EVALUATION

This section is organized as follows. First, the basic receiver equations are given in Section III-A. Then the SNR at the threshold comparator input is found in Section III-B. The BER is evaluated in Section III-C. Finally, in Section III-D numerical results are presented.

#### A. Basic Receiver Equations :

The dual filter optical FSK receiver under investigation is shown in Figure 1. Following [9] and [10], we assume the IF filters to be finite-time bandpass integrators, with impulse responses

$$h_{B1}(t) = \begin{cases} \frac{1}{T} \cos[\omega_1 t] & \text{if } t \in [0, \frac{T}{2}] \\ 0 & \text{if } t \notin [0, \frac{T}{2}] \end{cases} \quad (12)$$

and

$$h_{B2}(t) = \begin{cases} \frac{\alpha}{T} \cos[\omega_0 t] & \text{if } t \in [0, \frac{T}{\alpha}] \\ 0 & \text{if } t \notin [0, \frac{T}{\alpha}] \end{cases} \quad (13)$$

where  $\alpha \geq 1$  is a positive integer. The square-law device (SLD) is an envelope detector modeled by [9]

$$\begin{aligned} v_F(t) &\equiv |U_1(t)|^2 - |U_2(t)|^2 \\ &= x_F(t) + y_F(t) + z_F(t) \end{aligned} \quad (14)$$

where  $U_1(t)$  and  $U_2(t)$  are the complex amplitudes of the outputs of the IF filter 1 and IF filter 2, respectively. In expression (14),  $x_F(t)$ ,  $y_F(t)$ , and  $z_F(t)$  denote the signal-cross-signal, noise-cross-noise, and signal-cross-noise terms, respectively. The postdetection lowpass filter is assumed to have the following impulse response [9]:

$$h_L(t) = \sum_{i=1}^{\alpha} \delta(t - \frac{iT}{\alpha}) \quad (15)$$

The output current of the IF filters at time  $t$  is

$$\begin{aligned} u_1(t) &\equiv h_{B1}(t) * i_T(t) \\ &= s_{B1}(t) + n_{B1}(t) \quad lT \leq t \leq (l+1)T \end{aligned} \quad (16)$$

and

$$\begin{aligned} u_2(t) &\equiv h_{B2}(t) * i_T(t) \\ &= s_{B2}(t) + n_{B2}(t) \quad lT \leq t \leq (l+1)T \end{aligned} \quad (17)$$

where  $*$  denotes convolution;  $n_{B1}(t)$  and  $n_{B2}(t)$  are the filtered versions of the noise, and  $s_{B1}(t)$  and  $s_{B2}(t)$  are the filtered versions of the signal, respectively; i.e.,



$$n_{B1}(t) \equiv h_{B1}(t) * n(t) \quad lT \leq t \leq (l+1)T \quad (18)$$

$$n_{B2}(t) \equiv h_{B2}(t) * n(t) \quad lT \leq t \leq (l+1)T \quad (19)$$

and

$$s_{B1}(t) = h_{B1}(t) * A_S \cos[\omega_d t + \phi(t)], \quad lT \leq t \leq (l+1)T \quad (20)$$

$$s_{B2}(t) = h_{B2}(t) * A_S \cos[\omega_d t + \phi(t)], \quad lT \leq t \leq (l+1)T \quad (21)$$

where  $\omega_d = \omega_1$  or  $\omega_d = \omega_0$ . Hence the output of the summer is

$$v_F(t) \equiv x_F(t) + y_F(t) + z_F(t) \quad (22)$$

where

$$x_F(t) = s_{B1}^2(t) + \hat{s}_{B1}^2(t) - s_{B2}^2(t) - \hat{s}_{B2}^2(t) \quad (23)$$

$$y_F(t) = n_{B1}^2(t) + \hat{n}_{B1}^2(t) - n_{B2}^2(t) - \hat{n}_{B2}^2(t) \quad (24)$$

$$z_F(t) = 2[s_{B1}(t)n_{B1}(t) + \hat{s}_{B1}(t)\hat{n}_{B1}(t) - s_{B2}(t)n_{B2}(t) - \hat{s}_{B2}(t)\hat{n}_{B2}(t)] \quad (25)$$

In expressions (23)-(25),  $\hat{s}_{Bd}(t)$  is the Hilbert transform of  $s_{Bd}(t)$ ,  $\hat{n}_{Bd}(t)$  is the Hilbert transform of  $n_{Bd}(t)$ , and  $d = 0$  or  $d = 1$ . Finally, the output signal of the lowpass filter (LPF)  $w_F(t)$  is

$$w_F(t) = h_L(t) * v_F(t) \quad (26)$$

The signal  $w_F(t)$  is sampled at  $t = (l+1)T$  and sent to the decision gate. The value of  $w_F(t)$  at  $t = (l+1)T$  is denoted by  $w_{FT}$  (decision variable) in the remainder of this paper.

## B. The Signal-to-Noise Ratio :

The SNR at the input of the decision gate (Figure 1) is defined as [9], [18]

$$\gamma \equiv \frac{m(\text{when data} = 1) - m(\text{when data} = 0)}{\sigma(\text{when data} = 1) + \sigma(\text{when data} = 0)} \quad (27)$$

where  $m$  and  $\sigma$  are the mean and the standard deviation of  $w_{FT}$ . We note that, strictly speaking, the signal-to-noise ratio  $\gamma$  is a meaningful performance measure for a Gaussian hypothesis test only, whereas the probability density function (PDF) of  $w_{FT}$  is non-Gaussian for both ONE and ZERO symbols even when the linewidth is zero and the frequency deviation is infinite. In the case of zero linewidth and infinite frequency deviation, the LPF is not needed, and the bandwidth of the BPF is taken to be equal to the bit rate  $R_b$ . Conceptually, in this case  $w_F(t) = v_F(t)$ . The probability density function of  $v_F(t)$  in this case can be found exactly. When  $\Delta\nu$  is not equal to zero, and  $2\Delta f_d$  is finite, the impact of phase noise and crosstalk further complicates the PDF. Since the actual pdf of  $w_{FT}$  is non-Gaussian, it is clear that the value of  $\gamma$  does not contain all the information needed to evaluate the system BER exactly. Hence, using expression (27) for estimating the SNR at the decision gate is an approximation; see Subsection C and Ref.[9] for an analysis of the accuracy of this approximation.

Calculation of  $m$  and  $\sigma$  is a long and a very complicated process. Therefore, we omit the intermediate steps of our derivations. Our analysis indicates that a general closed-form result can be obtained for the signal-to-noise ratio at the decision gate:

$$\gamma = \frac{m_1}{\sigma_1} \quad (28)$$

where

$$m_1 = \frac{A_S^2}{4}(2\alpha) \left[ \frac{1}{a} + \frac{b-1}{a^2} + \frac{1}{k_1} \left\{ -a - \frac{(x^2 - a^2)}{k_1}(1 - b\cos x) + \frac{2abx\sin x}{k_1} \right\} \right] \quad (29)$$

and

$$\sigma_1^2 = \sigma_{XF}^2 + \sigma_{YF}^2 + \sigma_{ZF}^2 \quad (30)$$

In expression (30),  $\sigma_{XF}^2$ ,  $\sigma_{YF}^2$ , and  $\sigma_{ZF}^2$  are given by:

$$\sigma_{XF}^2 = 4 \frac{A_S^4}{16} \frac{\alpha}{a^4} \left[ a^2 - \frac{11}{2}a - \frac{16}{3}ab + \frac{80}{9}(1-b) + (1-b^2) - \frac{1}{72}(1-b^4) \right] s(2\Delta\omega T) \quad (31)$$

$$\sigma_{YF}^2 = \frac{A_S^4}{16} \frac{2\alpha^3}{\xi^2} \left[ 1 - \frac{\sin^2 x}{x^2} \right] \quad (32)$$

and

$$\begin{aligned} \sigma_{ZF}^2 = & \frac{A_S^4}{16} \frac{16\alpha^2}{\xi^2} \left[ -\frac{\sin^2 x}{4ax^2} + \frac{\sin x}{8xk_1} \left\{ 1 + \cos x + \left( \frac{x}{a} - \frac{2a}{x} \right) \sin x + (1-2b) \frac{y}{a} \right\} + \right. \\ & \left. \frac{a}{4k_1} + \frac{1}{4k_1^2} \left\{ [x^2 - a^2][1 - b\cos x] - 2abx\sin x \right\} + \frac{1}{4a} \left\{ 1 + \frac{1}{a}(b-1) \right\} \right] \end{aligned} \quad (33)$$

where

$$\tau = T/\alpha \quad (34)$$

$$a = \pi\Delta\nu\tau \quad (35)$$

$$b = \exp(-a) \quad (36)$$

$$x = 2\Delta\omega\tau \quad (37)$$

$$y = (a + a\cos x - x\sin x) \quad (38)$$

$$k_1 = (a^2 + x^2) \quad (39)$$

and

$$s(2\Delta\omega T) = \begin{cases} 1 - e^{-\beta\omega_n 2\Delta\omega T} \left[ \cos(q) + \frac{\beta}{\sqrt{1-\beta^2}} \sin(q) \right] & \text{for } \beta \neq 1 \\ 1 - e^{-\omega_n 2\Delta\omega T} - \omega_n 2\Delta\omega T e^{-\omega_n 2\Delta\omega T} & \text{for } \beta = 1 \end{cases} \quad (40)$$

In expression (40),  $\omega_n$ ,  $\beta$ , and  $q$  are defined as follows:

$$\omega_n \approx -0.0155 - 0.263/\alpha^2 + 0.578/\alpha \quad (41)$$

$$\beta \approx 0.27 - 1.5/\alpha^2 + 2.22/\alpha \quad (42)$$

$$q = \omega_n 2\Delta\omega T \sqrt{1 - \beta^2} \quad (43)$$

For the dual filter optical heterodyne FSK system under investigation, there is a simple relationship between the peak IF SNR  $\xi$ , peak normalized signal energy  $E_{pn}$ , and the average normalized signal energy per bit  $E_{an}$  defined as

$$\xi = E_{pn} = E_{an} = \frac{A_S^2 T}{2\eta} \quad (44)$$

The value of  $\gamma$  in expressions (28)-(43) can be considerably simplified in several important practical cases:

**Case 1:** If  $\Delta\nu = 0$  (no phase noise), then  $\gamma$  reduces to

$$\gamma = \frac{\sqrt{2\alpha} \left( \frac{1}{2} - \frac{(1 - \cos x)}{x^2} \right)}{\left[ \frac{\alpha^2}{\xi^2} \left( 1 - \frac{\sin^2 x}{x^2} \right) + \frac{8}{\xi} \left( \frac{\cos^2 x - \cos x}{4x^2} + \frac{1}{8} \right) \right]^{0.5}} \quad (45)$$

Expression (45) predicts the performance of the FSK system under consideration when there is no phase noise. Note that in this simple expression  $x = 2\Delta\omega T$  is a design parameter. Hence, to obtain a BER =  $10^{-9}$  (which corresponds to  $\gamma = 6$ ), the IF filter bandwidth expansion factor  $\alpha$  can be optimized for fixed values of  $2\Delta\omega T$ . By using  $\alpha_{opt}$  values in expression (45), the impact of reducing  $2\Delta\omega T$  on the sensitivity (the value of average signal energy in photons/bit) can be easily computed. As a sanity check, we also note that as  $2\Delta\omega T$  gets sufficiently large, expression (45) reduces for  $\alpha_{opt} = 1$  to

$$\gamma = \frac{1}{\sqrt{2 \left( \frac{1}{\xi^2} + \frac{1}{\xi} \right)}} \quad (46)$$

which is the result given in [9].

**Case 2:** If  $\frac{\Delta\nu T}{\alpha} \ll 1 \ll \frac{2\Delta\omega T}{\alpha}$ , then  $\gamma$  becomes

$$\gamma \approx \frac{\sqrt{2\alpha} \left( \frac{1}{2} - \frac{\pi\Delta\nu T}{6\alpha} - \frac{(1 + \frac{\pi\Delta\nu T}{\alpha} \cos(2\Delta\omega T/\alpha))}{(2\Delta\omega T/\alpha)^2} \right)}{\left[ \frac{2}{45} (\pi\Delta\nu T/\alpha)^2 s(2\Delta\omega T) + \frac{\alpha^2}{\xi^2} + \frac{2\alpha}{\xi} \left( \frac{1}{2} - \frac{\pi\Delta\nu T}{6\alpha} - \frac{(1 + \frac{\pi\Delta\nu T}{\alpha} \cos(2\Delta\omega T/\alpha))}{(2\Delta\omega T/\alpha)^2} \right) \right]^{0.5}} \quad (47)$$

Expression (47) predicts the SNR at the decision gate of a well-designed dual filter heterodyne FSK lightwave system in presence of laser phase noise and finite frequency deviation. We note that for a well-designed system,  $\Delta\nu T \ll \alpha \ll 2\Delta\omega T$ . Clearly, expression (47) is an approximation for the SNR at the decision gate. The inaccuracy of expression (47) compared to the exact SNR results given by expressions (28)-(43), however, is only 0.1 dB for a well-designed system. In Table I, simple guidelines are provided for the choice of  $\alpha_{opt}$  and  $2\Delta f_d T$  for a given  $\Delta\nu$  in well-designed systems.

### C. The Bit Error Rate

A simple estimate of the BER can be obtained by using the Gaussian approximation [9], [18]

$$BER \equiv Q(\gamma) = \frac{1}{\sqrt{2\pi}} \int_{\gamma}^{\infty} e^{-x^2/2} dx \quad (48)$$

We note that expression (48) gives the accurate value of BER for a Gaussian hypothesis test only and assumes that the decision threshold is selected optimally, i.e., at the intersection of the conditional pdf's of  $w_{FT}$  calculated for  $data = 1$  and for  $data = 0$ . As mentioned before, in the problem investigated, the pdf of  $w_{FT}$  is generally non-Gaussian. It was shown in [9] that using Gaussian PDF's for BER evaluation leads to an inaccuracy of approximately 2.6 dB. This, however, is not a serious limitation in itself since it can be somewhat rectified using an empirical factor. Such an empirical factor was shown to be useful in estimating the BER; in particular, it was shown that the inaccuracy of the BER predictions of Gaussian approximation can be reduced to less than 1 dB over a wide range of linewidth values (up to 250%) when such an empirical correction factor is used. Hence, in this paper we estimate BER as

$$BER \equiv Q(\gamma') = \frac{k}{\sqrt{2\pi}} \int_{\gamma}^{\infty} e^{-x^2/2} dx \quad (49)$$

where  $k = 1.36$  is the empirical factor used for reducing the inaccuracy of the Gaussian BER estimate to 0 dB at  $\Delta\nu T = 0$ .

It should be noted that though the absolute accuracy of the Gaussian approximation technique is not perfect, a modified Gaussian approximation does provide a very useful analytical tool for studying the impact of crucial system parameters such as finite frequency deviation and laser linewidth on system performance.

#### D. Numerical Results

Figure 3 shows the optimum IF filter bandwidth values  $\alpha_{opt}$  versus normalized laser linewidth for fixed values of the frequency separation  $2\Delta f_d T$ . We observe that for a fixed linewidth, the required  $\alpha_{opt}$  decreases as the frequency deviation becomes smaller. The physical reason of this phenomenon is the so-called crosstalk. As the frequency deviation gets narrower, the IF filter collects more crosstalk from the other signal. To maintain the same BER (let's say  $BER = 10^{-9}$ ) as in the case of infinite frequency deviation, a narrower IF filter bandwidth must be used in order to reduce the influence of crosstalk. There is, however, a certain IF filter bandwidth value beyond which a further reduction in IF filter bandwidth is detrimental because the information-bearing signal gets substantially truncated and the loss in signal power becomes more than the loss in noise power. This tendency can easily be observed in Figure 4(a). Figure 4(b) shows  $\alpha_{opt}$  versus normalized frequency deviation for several linewidths at  $BER = 10^{-9}$ . A careful inspection of Figure 4(b) reveals that, at each fixed linewidth there is a required  $\alpha_{opt}$  and  $2\Delta f_d T$  for maintaining a  $BER = 10^{-9}$ . We emphasize that Figure 4(b) does not necessarily correspond to a well-designed system. When  $\Delta\nu T = 1.0$ , for example, Figure 4(b) indicates that both  $\alpha_{opt}$  and  $2\Delta f_d T$  can be as small as 3 and a  $BER = 10^{-9}$  can still be maintained. Such a system, however, is not a well-designed system. It is clear from Table I that,

when  $\Delta\nu T = 1.0$ ,  $\alpha_{opt} = 6$  and  $2\Delta f_d T \approx 5.46$  in a well-designed system.

Figures 5-9 show BER versus  $\xi$  for fixed linewidth values and for  $\alpha_{opt}$  as the frequency deviation varies parametrically. As expected, for a fixed linewidth value, the sensitivity of the system deteriorates as frequency deviation becomes smaller. As an example, Figure 7 shows that, when  $2\Delta f_d = 0.75R_b$  the sensitivity of the system is 3 dB worse than the ideal case ( $2\Delta f_d = \infty$ ). Note that (see Figures 3 and 4(a))  $\alpha_{opt} = 1$  for  $2\Delta f_d = 0.75R_b$  as opposed to  $\alpha_{opt} = 4$  in the ideal case when  $\Delta\nu T = 0.16$ . The curves shown in Figures 5-9 are plotted using  $\alpha$  values which are optimum only at  $BER = 10^{-9}$ . Hence, for  $BER \neq 10^{-9}$  the curves are not optimum. This is the main reason for the better performance of the  $2\Delta f_d = 0.75R_b$  curve compared to the  $2\Delta f_d = 1.5R_b$  curve at BER values larger than  $10^{-9}$  in Figure 8.

In Figures 10-14 the BER performance is shown versus  $\xi$  for a fixed frequency deviation value as the laser linewidth increases. It can be observed from these Figures that for a fixed frequency deviation, there is a finite degradation in sensitivity (or an increase in the required  $\xi$  for  $BER = 10^{-9}$ ) as linewidth becomes larger. Figure 12, for instance, shows that there is approximately a 2.5 dB sensitivity penalty when the linewidth increases from 0 to 16%, for a fixed frequency deviation of  $2\Delta f_d = 1.5R_b$ . Another important result which should be observed from Figures 10-14 is that the aforementioned effect becomes stronger for decreasing values of frequency deviation.

In Figures 15-17 the sensitivity penalty (in dB) is plotted versus laser linewidth for fixed frequency deviation values. Figure 15, for example, clearly illustrates that for a fixed linewidth value the sensitivity penalty increases as the frequency separation between the two tones decreases. For a linewidth of 50%, Figure 15 shows that the sensitivity penalty becomes more than 5 dB for a frequency separation of  $2\Delta f_d = R_b$  which is a drastic deterioration compared to 0.88 dB penalty in the ideal case.

Figure 16 shows the sensitivity penalty of the system under investigation versus frequency deviation  $2\Delta f_d T$  for several laser linewidths when optimum IF filter bandwidth values are employed. It

can be observed from Figure 16 that, for a 1 dB sensitivity penalty the required frequency deviation is  $\approx 2\Delta f_d = 0.72R_b$  for zero linewidth whereas this number goes up to  $2\Delta f_d = 3.4R_b$  for 50% linewidth. We emphasize that in Figure 16 the  $\alpha$  values used are always optimum. It is interesting to contrast the results of Figure 16 with those where  $\alpha$  values are not optimum. This is illustrated in Figure 17. In Figure 17, each sensitivity penalty curve uses the  $\alpha$  value which is optimum only for the ideal case ( $\alpha_{inf-opt}$ ); i.e., when  $2\Delta f_d T = \infty$ . It can be easily seen that suboptimum  $\alpha$  values will make the frequency deviation requirement for 1 dB penalty worse.

#### IV. PHYSICAL INTERPRETATION OF RESULTS

The performance of the dual-filter heterodyne FSK lightwave receiver under investigation depends on the IF laser linewidth, frequency deviation between the two frequencies of data transmission, IF, and the bandwidth of the two IF filters. In this study we assume that IF is sufficiently large so that the sensitivity penalty due to finite IF is negligible. This assumption also enables us to isolate the impact of finite frequency deviation on system performance in presence of phase noise and shot noise.

In this paper it is shown that the frequency deviation, laser linewidth, and the optimum IF filter bandwidths are interrelated system parameters. Furthermore, this relationship is quantified by the results of our analysis.

It is interesting to note that, though they stem from different physical origins, the net impact of finite frequency deviation on system performance is very similar to the impact of laser linewidth on system performance. For each linewidth and frequency deviation value, there is an optimum IF filter bandwidth. If for that fixed linewidth value the frequency deviation is decreased, more crosstalk power is collected by each IF filter. For maintaining the same BER, a smaller IF filter bandwidth is required. When the IF filter bandwidth is reduced, however, the information-bearing signal is also truncated. For each linewidth there is a critical value of the IF filter bandwidth beyond which a



further reduction in bandwidth implies more loss in signal power than noise power and, therefore, a severe BER floor. Hence, for optimum system performance the IF filter bandwidth should be optimized for each linewidth value and frequency deviation. If this is not done and the optimum IF filter bandwidth for the ideal case (i.e. if  $\alpha = \alpha_{inf-opt}$ ) is used for all frequency deviation values, the sensitivity penalty paid rises sharply as the frequency deviation decreases. This result is illustrated in Figure 18 for a linewidth of 27%. In other words, larger-than-optimum IF filter bandwidth values may have a very profound adverse effect on system performance. The physical reason for this degradation is the excess crosstalk power in addition to the excess shot noise collected by the IF filters when the bandwidths used are larger-than-optimum.

Similarly, for a fixed frequency separation  $2\Delta f_d T$ , the optimum IF filter bandwidths required increase as the linewidth increases. In other words, in order to maintain a  $BER = 10^{-9}$  larger IF filter bandwidths are required for larger linewidth values. Therefore, if one wants to pay a 1 dB sensitivity penalty due to laser linewidth at a fixed value of frequency deviation, it is clear that there is a maximum permissible laser linewidth. Beyond this  $\Delta\nu$  value, the sensitivity penalty is more than 1 dB. This phenomenon is a result of the interaction of the influence of crosstalk due to finite frequency deviation with the influence of phase noise. Specifically, for larger values of linewidth a larger value of  $\alpha_{opt}$  is required. As  $\alpha_{opt}$  gets larger, however, the IF filters collect more crosstalk and the sensitivity deteriorates.

The sensitivity penalty curves shown in Figures 16 and 17 also reveal an important physical phenomenon. When the linewidth is zero, for example, the sensitivity penalty is zero at different frequency deviation values; namely, at integer multiples of the bit rate. These values of frequency deviation which give equivalent performance with infinite frequency deviation are known as orthogonal values. Similarly, a dual filter FSK system which uses integer multiples of bit rate for the frequency deviation between the two information frequencies is known as an orthogonal FSK system. Figure 16 clearly shows that, in such a system the sensitivity penalty has an oscillatory behaviour

as frequency deviation becomes smaller, up to a certain frequency deviation (up to  $2\Delta f_d = R_b$ ). Hence, surprisingly, the system performance for  $2\Delta f_d = R_b$ , for example, is actually better than the system performance at  $2\Delta f_d = 2.5R_b$ . Note that the sensitivity penalty increases uniformly and sharply for  $2\Delta f_d < R_b$ . A simple mathematical derivation for the performance of the dual-filter FSK system with zero linewidth is given in [21]. That derivation gives some insight into the reasons of the mentioned oscillatory behaviour. It can be shown that when  $f_1$  and  $f_0$  are orthogonal the two signals are uncorrelated in the signal-space constellation diagram. Therefore, as long as the system is orthogonal its performance is not affected by the actual value of the frequency deviation. For non-negligible linewidths, Figure 17 shows the same type of oscillatory behaviour for sensitivity penalty. Note, however, that the ideal performance is never reached at any finite frequency deviation value for non-negligible linewidths. In other words, the dual-filter FSK lightwave system is never orthogonal in the true sense for finite (non-negligible) linewidths.

The sensitivity penalty results shown in Figures 16 and 17 can be checked with the classical communication theory literature for zero linewidth [19]-[22]. In all these references, it is a well established fact that the crosstalk is negligible when  $2\Delta f_d = R_b$ . This gives us a certain confidence in the validity of the approach used in this paper. For non-negligible linewidths, however, it remains to be seen how close the results predicted by our approximate theory are to those which will be predicted by a much more exact approach such as [10].

Though in this paper a single channel dual-filter FSK lightwave system is studied, the results obtained can give very rough estimates about the electrical domain channel spacings required in a multichannel dual-filter FSK system. In such a system, clearly there will be adjacent channel interference in addition to the co-channel interference due to finite frequency deviation.

## V. CONCLUSIONS

In this paper the impact of finite frequency deviation on the performance of dual-filter FSK lightwave systems in presence of laser phase noise and shot noise is studied and quantified. Our analysis shows that for zero linewidth, a frequency deviation of  $2\Delta f_d = 0.72R_b$  is required for 1 dB penalty. When the linewidth is 50%, for 1 dB sensitivity penalty the required frequency deviation increases to  $2\Delta f_d = 3.4R_b$ . In this work we also show that for a fixed linewidth, the optimum IF filter bandwidth decreases as the frequency deviation becomes narrower. As an example, for  $\Delta\nu = 0.5R_b$ ,  $\alpha_{opt}$  reduces from 7 to 3 as  $2\Delta f_d$  reduces from very large values to  $1.5R_b$ . The BER computations carried out in this paper enable us to estimate the largest permissible linewidth values for a fixed frequency deviation. One of the main strengths of this analysis is the fact that it leads to a simple closed-form SNR expression in terms of the main system parameters. This closed-form simple expression facilitates physical insight into the impact of finite frequency deviation on system performance and provides simple guidelines for practical system design.

The results obtained in this paper are based on the Gaussian assumption for the distributions of the two decision variables. We checked the results of our analysis for zero linewidth with the results given by classical communication theory and found a good match.

Factors not taken into account may change the results obtained in this paper. Receiver thermal noise and finite IF are among such factors. The ideal case of  $f_{IF} \gg R_b$  was assumed in this study in order to isolate the impact of finite frequency deviation. Further study is needed, however, to quantify the impact of finite IF (in presence of laser phase noise, finite frequency deviation, and shot noise) on the system performance of the dual-filter FSK lightwave systems.

The obtained results should be conceived as essential first steps for computing the electrical domain and optical domain channel spacing required for a prescribed sensitivity penalty in multichannel dual-filter FSK lightwave systems.

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**Table 1: Requirements for a well-designed receiver**

|                                 |               |   |
|---------------------------------|---------------|---|
| $\Delta\nu T \ll \alpha$        | $\Rightarrow$ | $0.29 + 5.95(\Delta\nu T) - 0.68(\Delta\nu T)^2 \leq [\alpha]$                          |
| $\alpha \ll 2\Delta f_d T$      | $\Rightarrow$ | $\alpha \leq 1.073(2\Delta f_d T)$  |
| $\Delta\nu T \ll 2\Delta f_d T$ | $\Rightarrow$ | $0.9 + 4.57(\Delta\nu T) + 1.9(\Delta\nu T)^2 - 1.91(\Delta\nu T)^3 \leq 2\Delta f_d T$ |

## Figure Captions :

**Fig. 1** Block diagram of the dual-filter heterodyne FSK system under investigation.

**Fig. 2** Physical scenario under consideration :

- (a) The frequency spectrum of the two information-bearing signals with infinitely large frequency separation.
- (b) The same frequency spectrum with finite frequency separation.
- (c) The frequency spectrum of the two information-bearing signals with finite frequency separation and zero phase noise.
- (d) The spectrum in (c) with the same frequency separation and non-negligible phase noise.

**Fig. 3** Optimum normalized IF filter bandwidth versus normalized linewidth for several values of frequency deviation at  $\text{BER} = 10^{-9}$ .

**Fig. 4(a)** Optimum normalized IF filter bandwidth versus normalized linewidth for several values of frequency deviation at  $\text{BER} = 10^{-9}$ .

**Fig. 4(b)** Optimum normalized IF filter bandwidth versus normalized frequency deviation for several linewidths at  $\text{BER} = 10^{-9}$ .

**Fig. 5** BER versus IF SNR for several values of frequency deviation when the linewidth is zero.

**Fig. 6** BER versus IF SNR for several values of frequency deviation when  $\Delta\nu T = 0.04$ .

**Fig. 7** BER versus IF SNR for several values of frequency deviation when  $\Delta\nu T = 0.16$ .

**Fig. 8** BER versus IF SNR for several values of frequency deviation and when  $\Delta\nu T = 0.27$ .

**Fig. 9** BER versus IF SNR for several values of frequency deviation and  $\Delta\nu T = 0.50$ .



**Fig. 10** BER versus IF SNR for several linewidths and infinite frequency separation between  $f_1$  and  $f_0$ .

**Fig. 11** BER versus IF SNR for several linewidths when  $2\Delta f_d = 4.5R_b$ .

**Fig. 12** BER versus IF SNR for several linewidths when  $2\Delta f_d = 1.5R_b$ .

**Fig. 13** BER versus IF SNR for several linewidths when  $2\Delta f_d = 0.75R_b$ .

**Fig. 14** BER versus IF SNR for several linewidths when  $2\Delta f_d = 0.3R_b$ .

**Fig. 15** Sensitivity penalty (in dB) versus normalized linewidth for several values of frequency deviation at  $\text{BER} = 10^{-9}$ .

**Fig. 16** Sensitivity penalty (in dB) versus normalized frequency deviation for several linewidths at  $\text{BER} = 10^{-9}$ . All curves are computed using  $\alpha = \alpha_{opt}$  at  $\text{BER} = 10^{-9}$ .

**Fig. 17** Sensitivity penalty (in dB) versus normalized frequency deviation for several linewidths at  $\text{BER} = 10^{-9}$ . All curves are computed using the optimum  $\alpha$  values for infinite frequency separation ( $\alpha_{inf-opt}$ ).

**Fig. 18** Impact of suboptimum  $\alpha$  values on the sensitivity penalty for  $\Delta\nu T = 0.27$  at  $\text{BER} = 10^{-9}$ .

The solid line corresponds to the curve computed with  $\alpha_{opt}$  for all frequency deviation values and the broken line corresponds to the curve computed with  $\alpha_{inf-opt}$ .

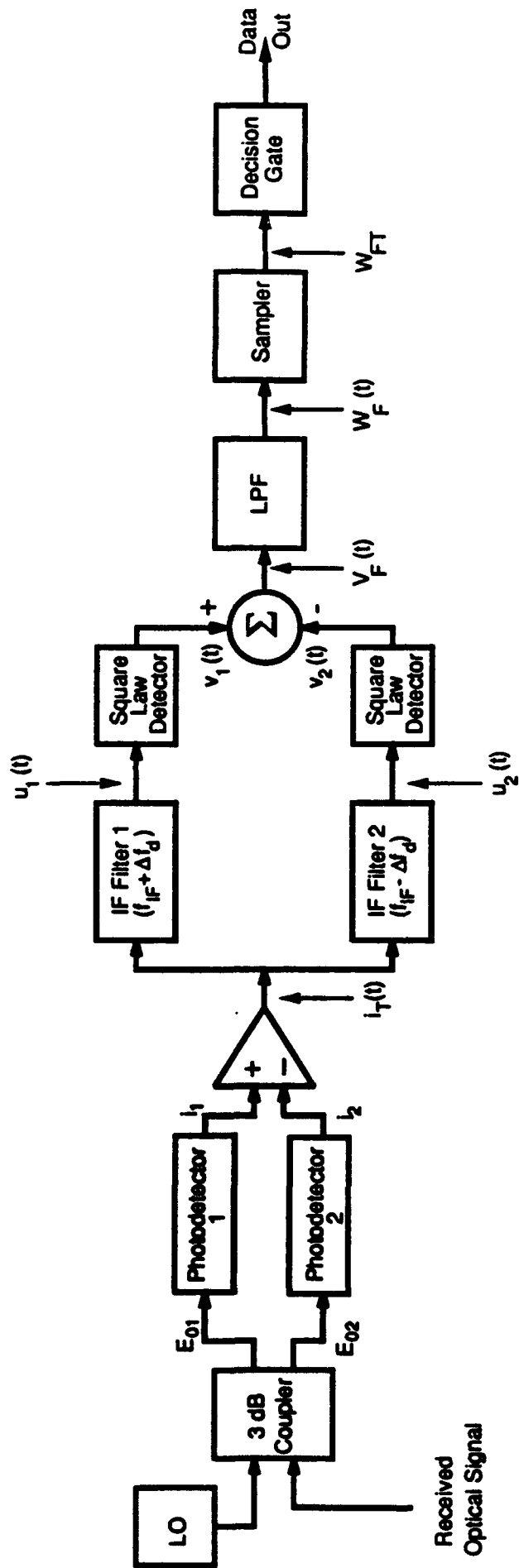
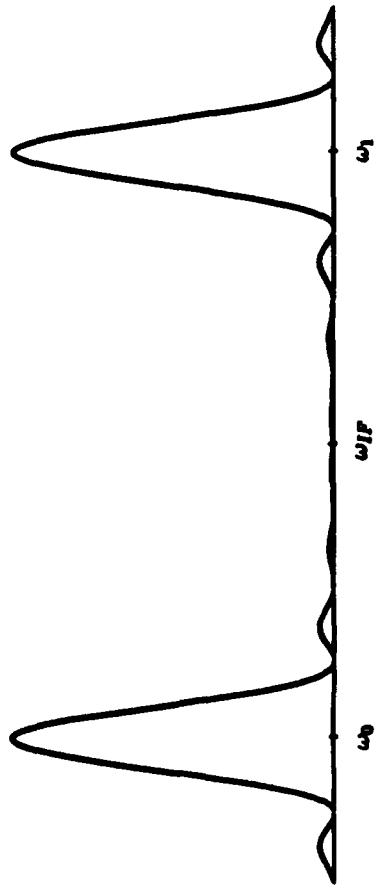
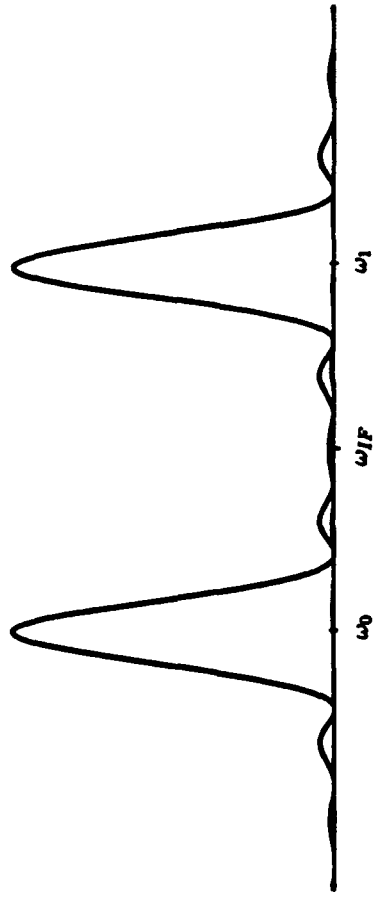


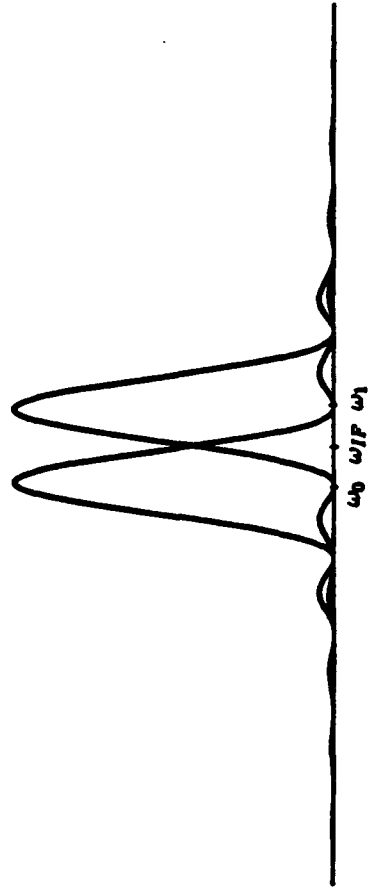
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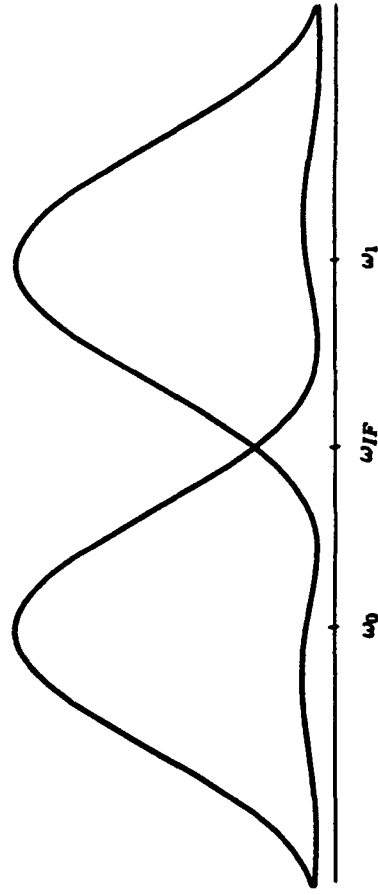
(a)



(c)



(b)



(d)

Figure 2

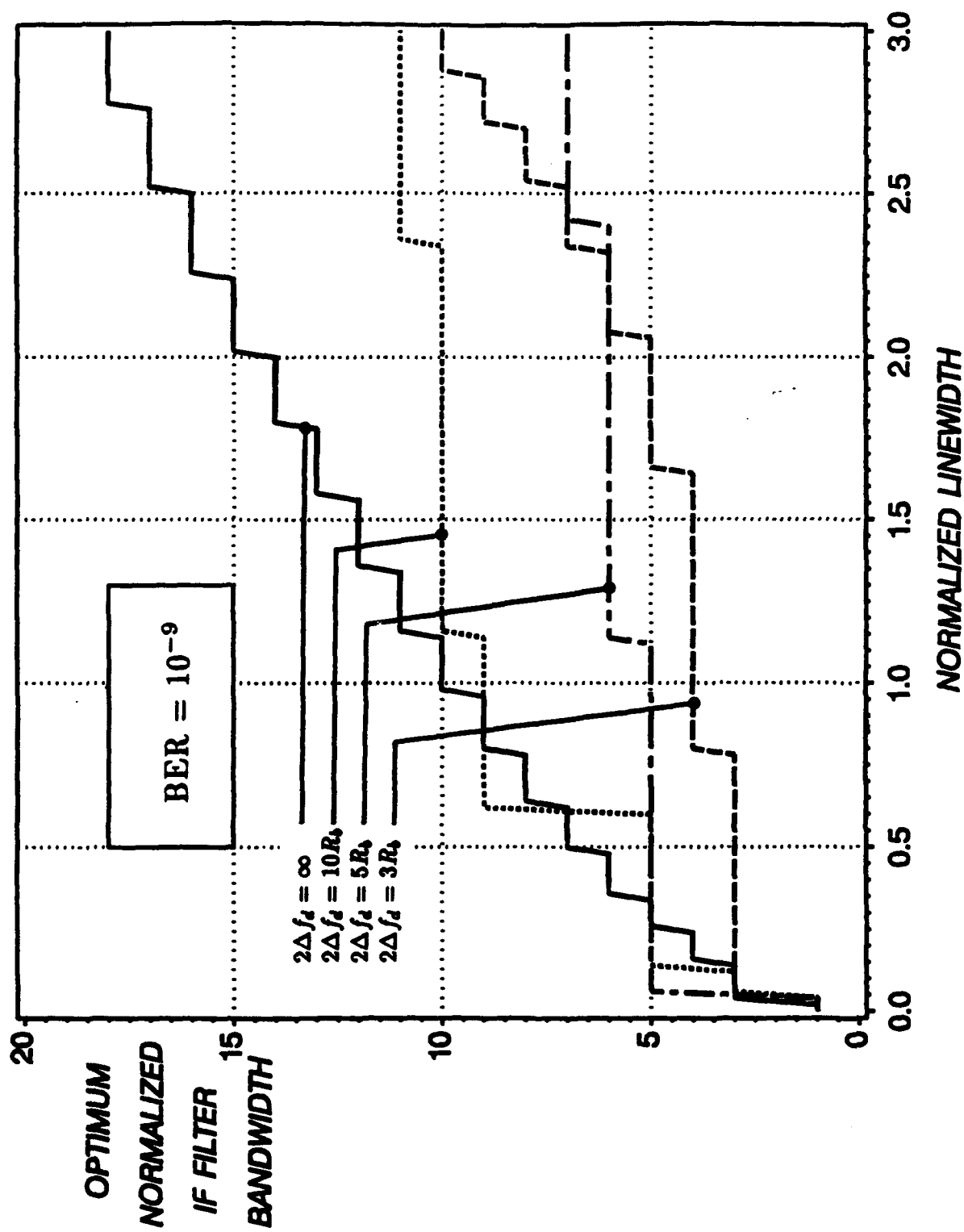


Figure 3

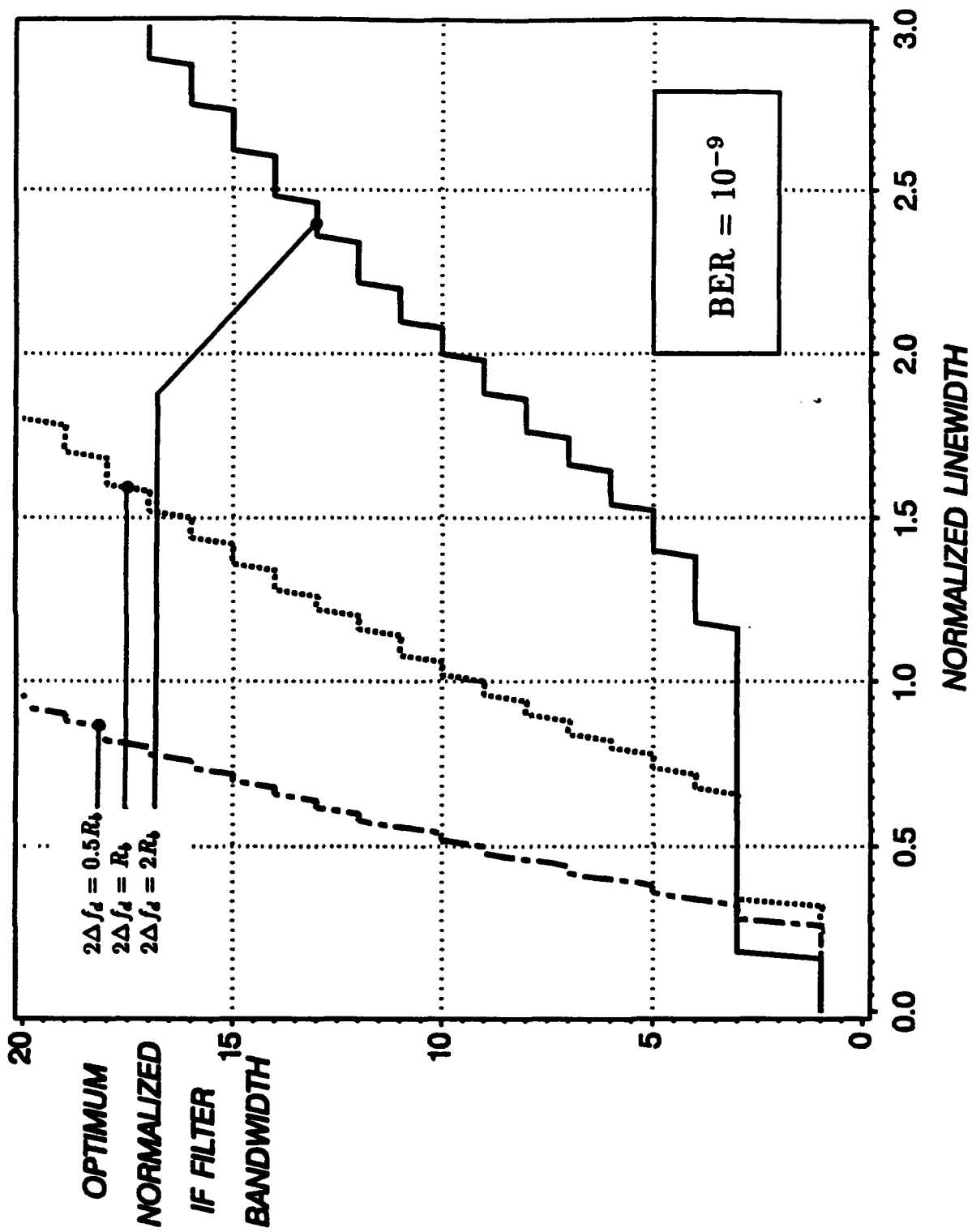


Figure 4(a)

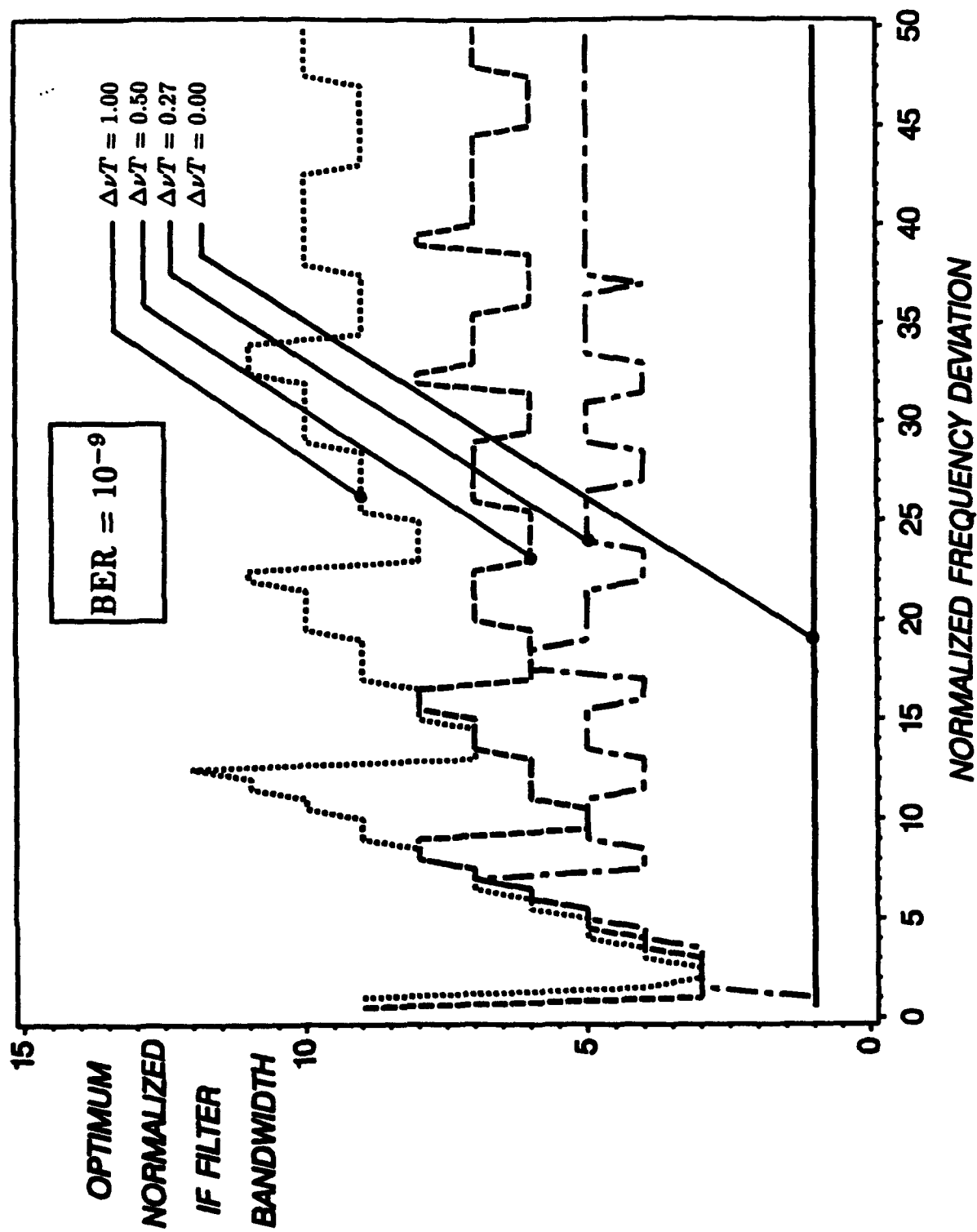


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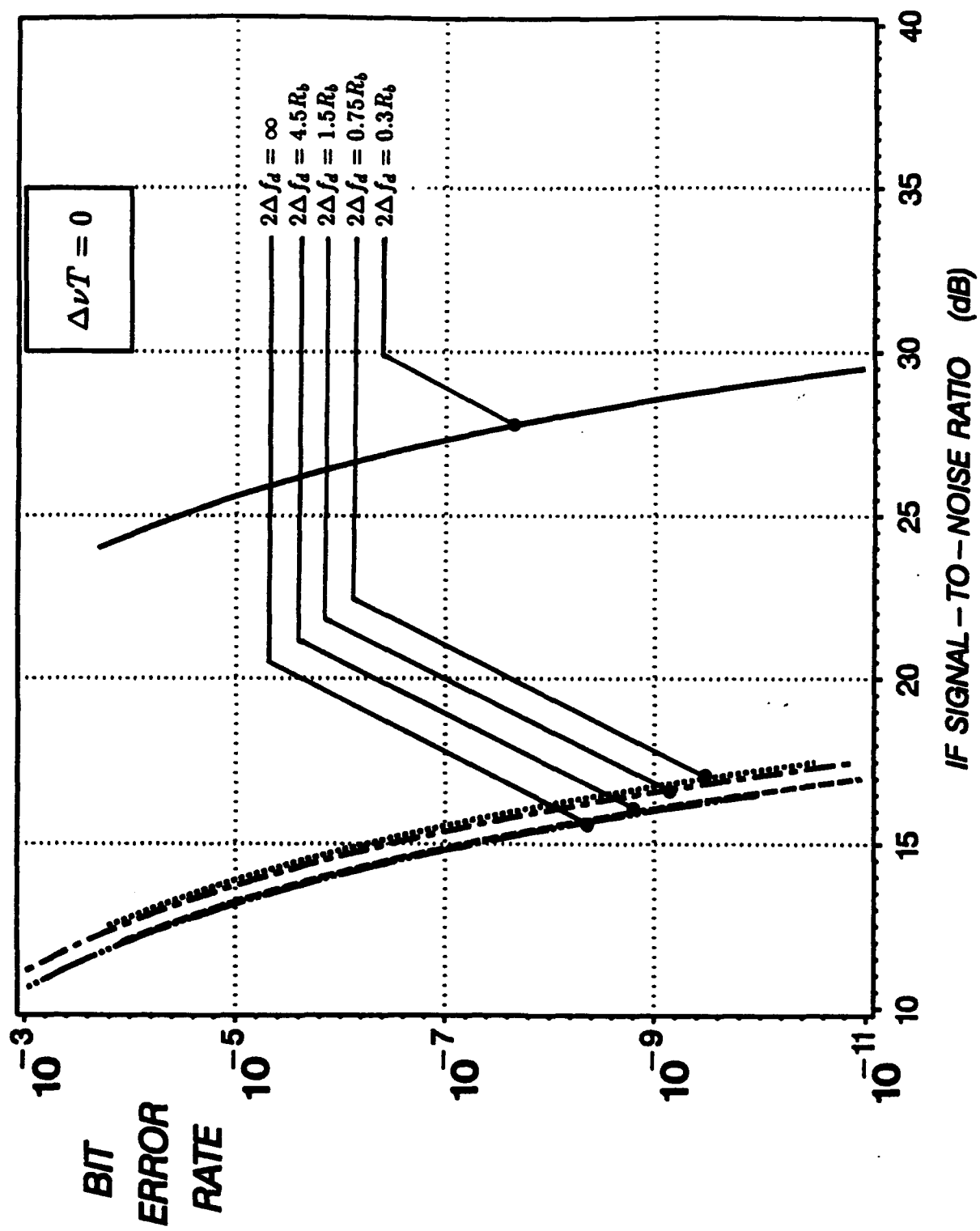


Figure 5

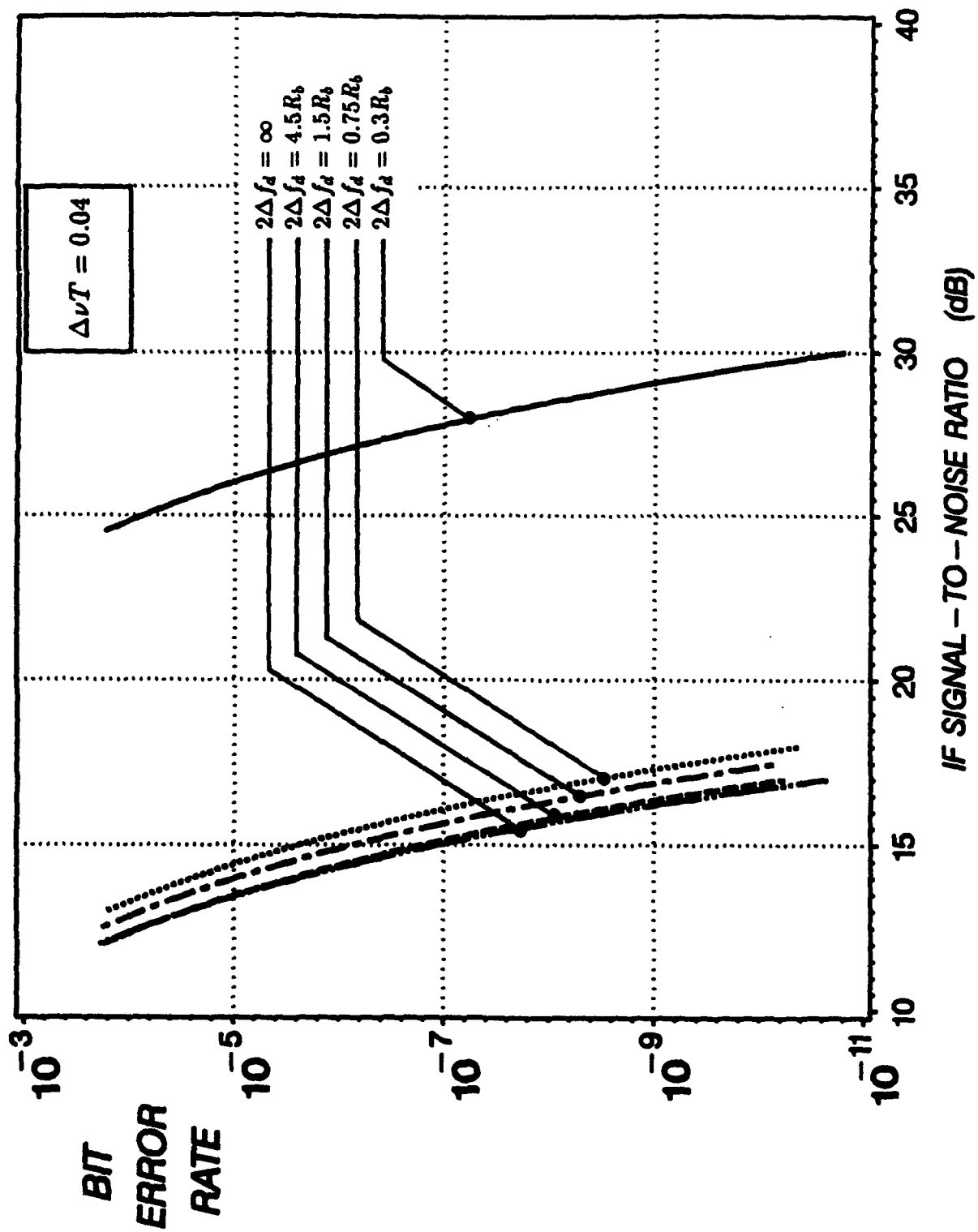


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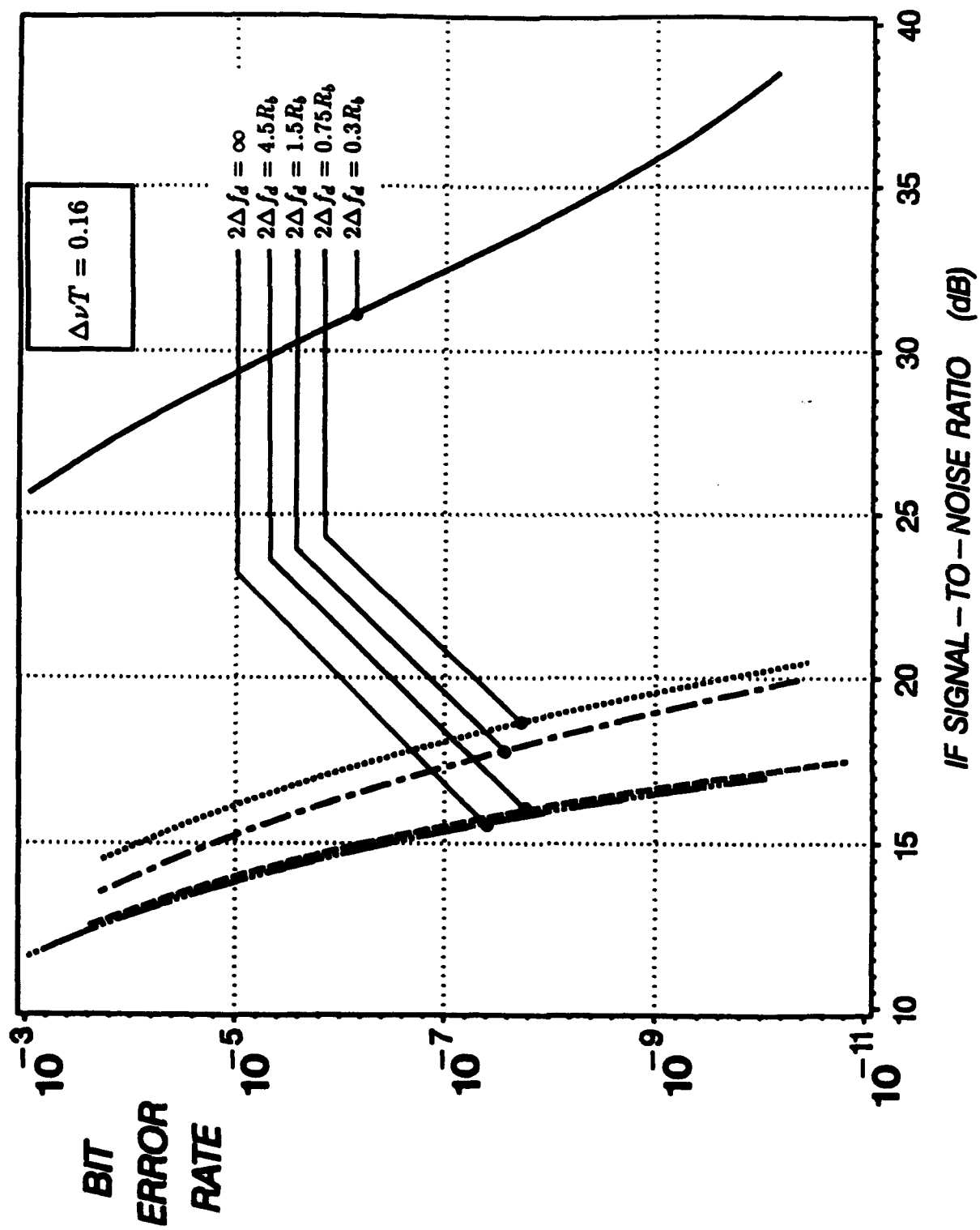


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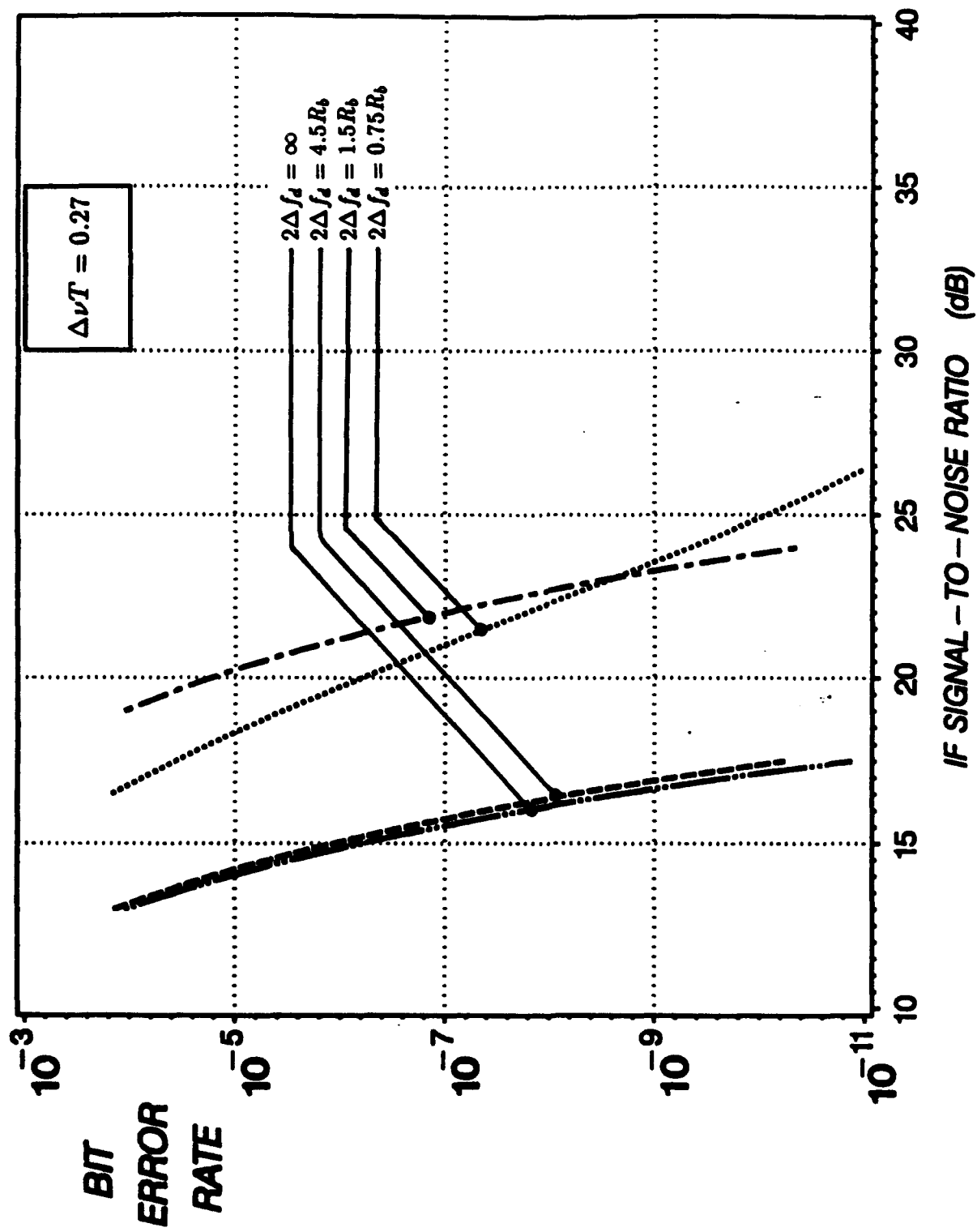


Figure 8

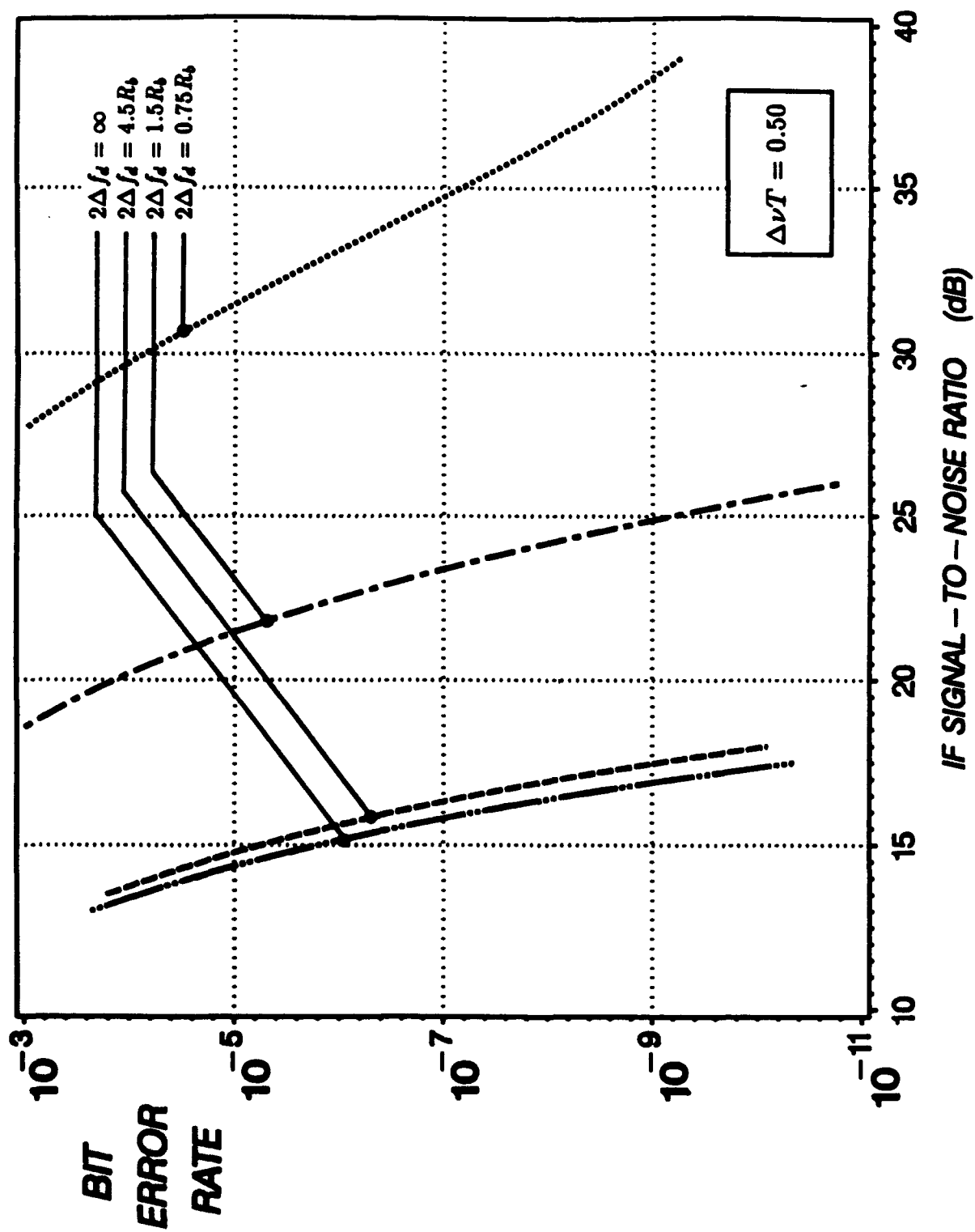


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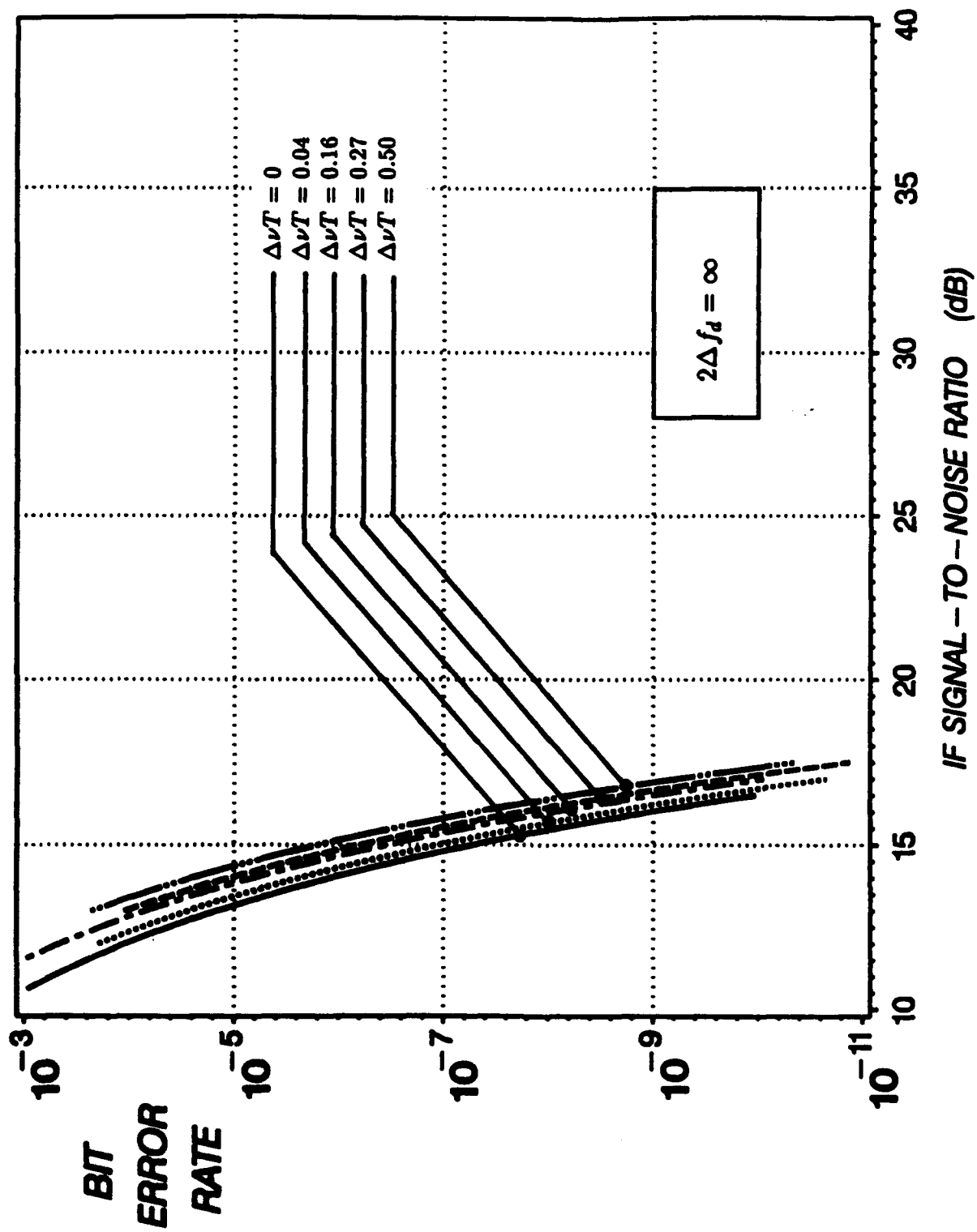


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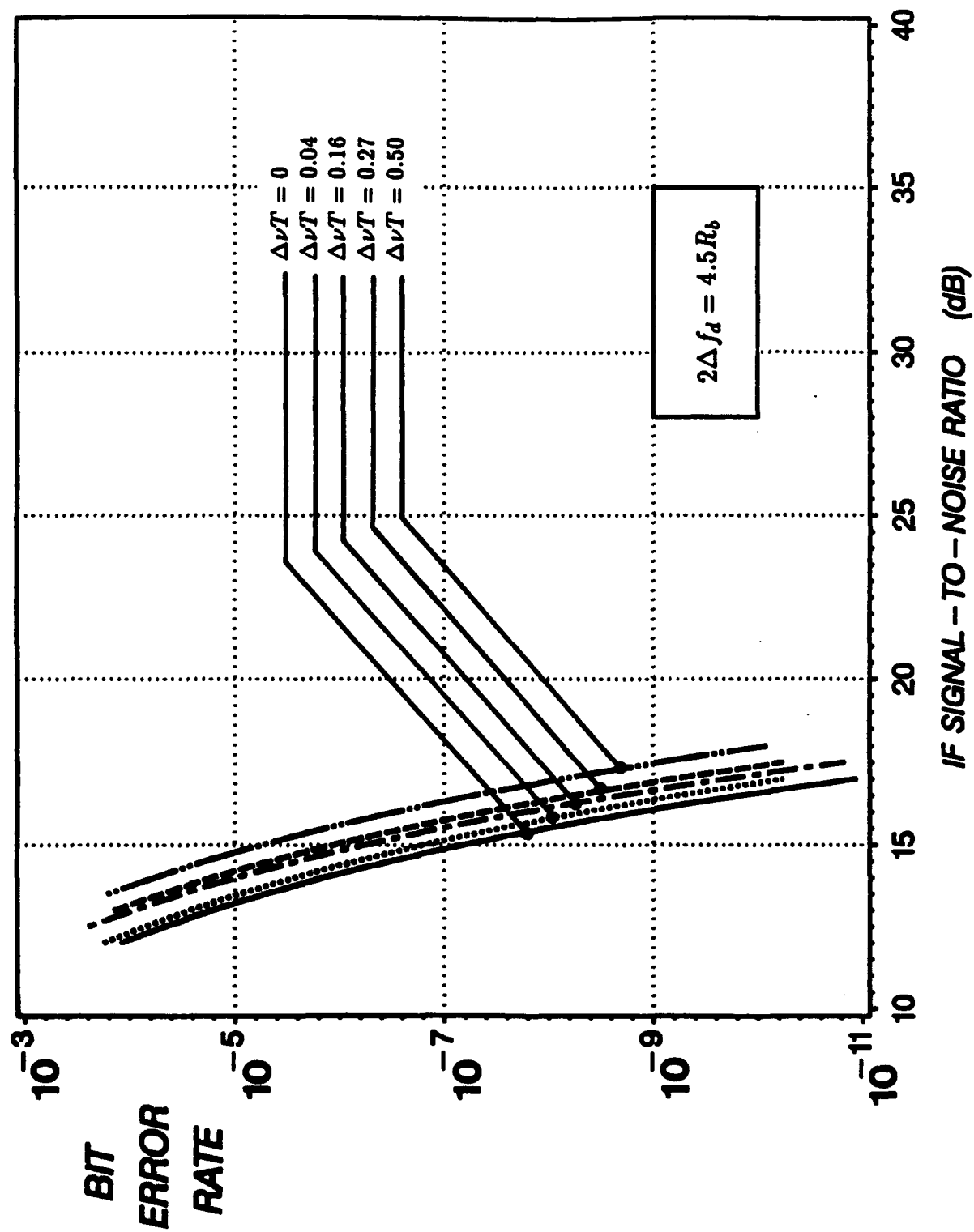


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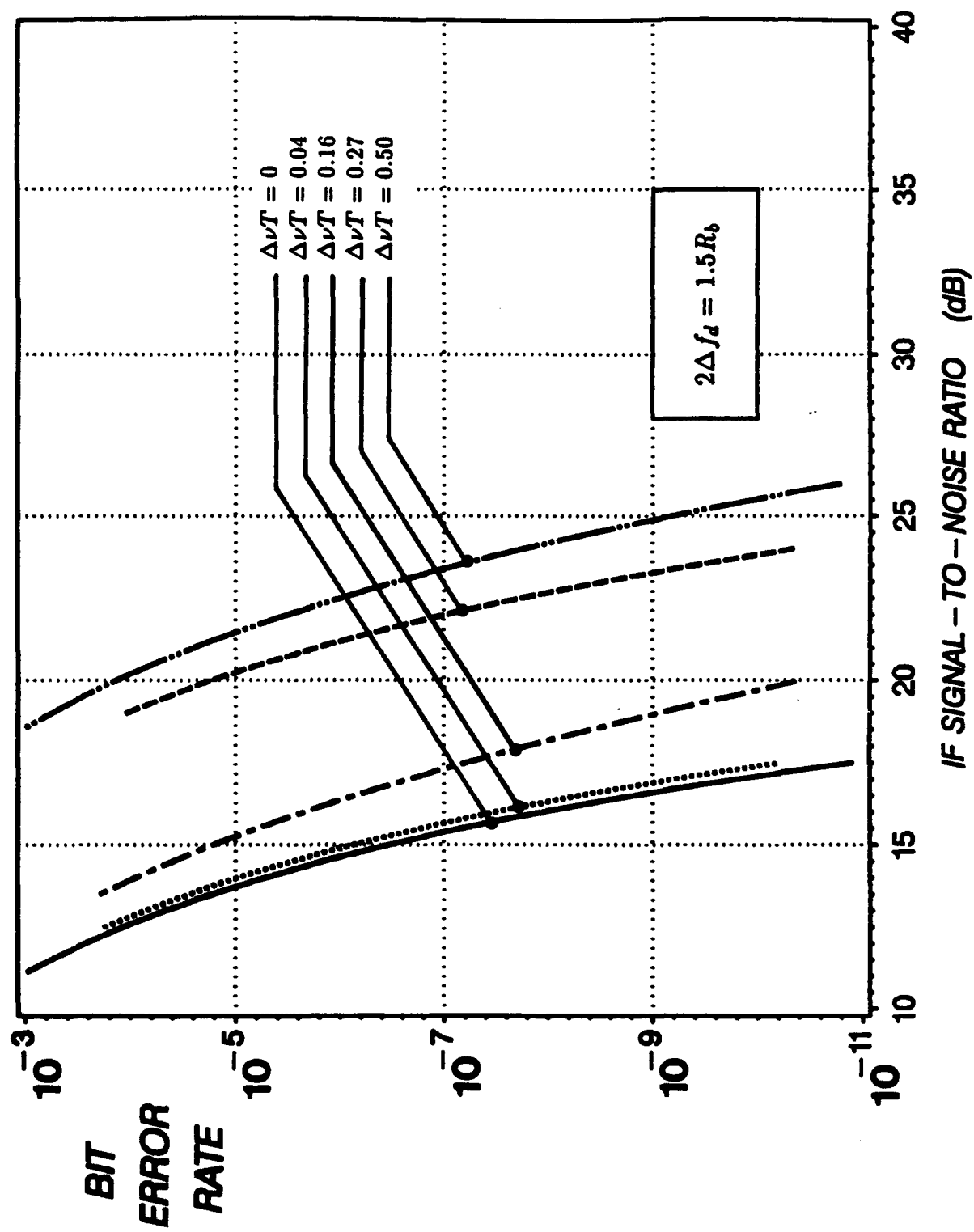


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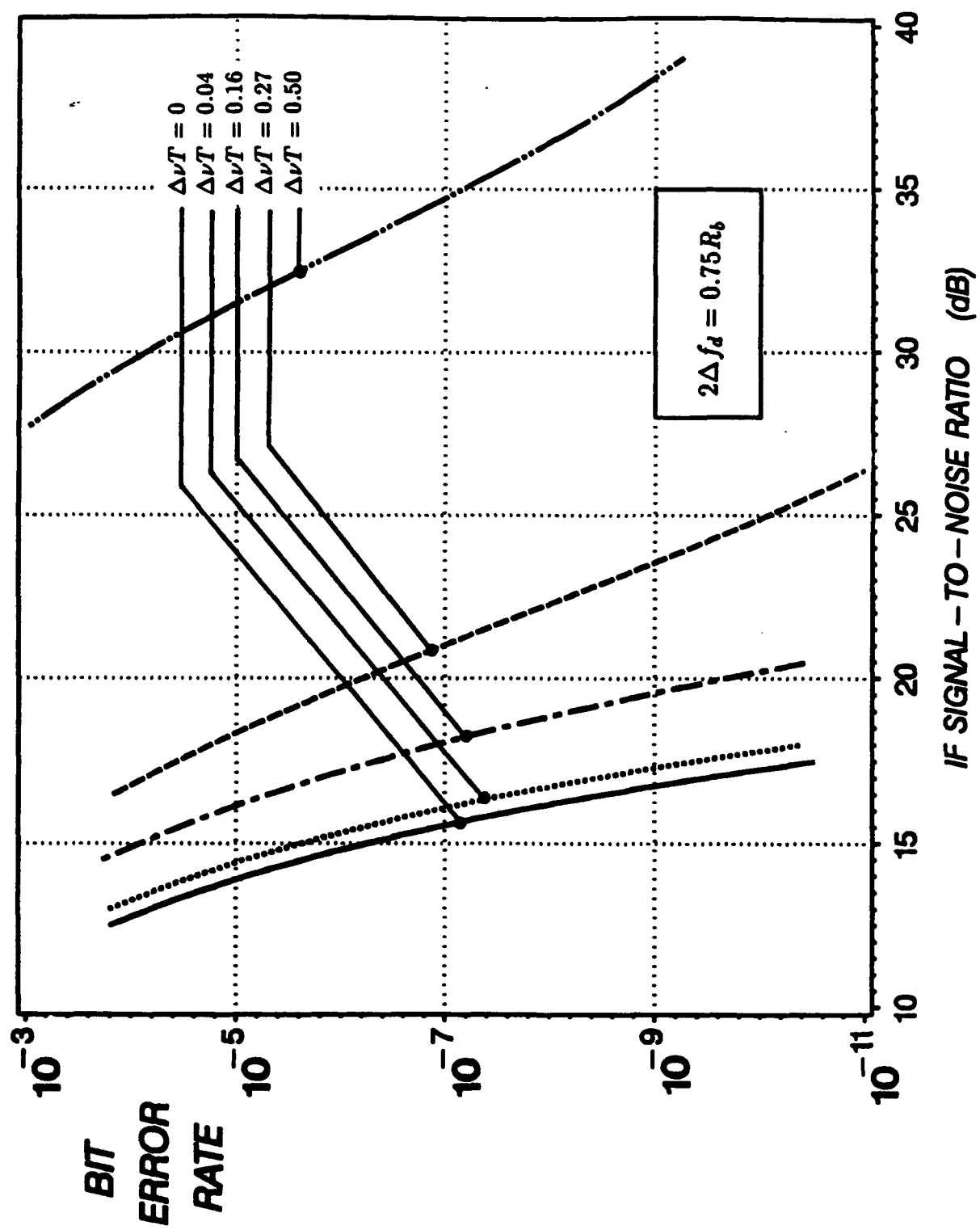


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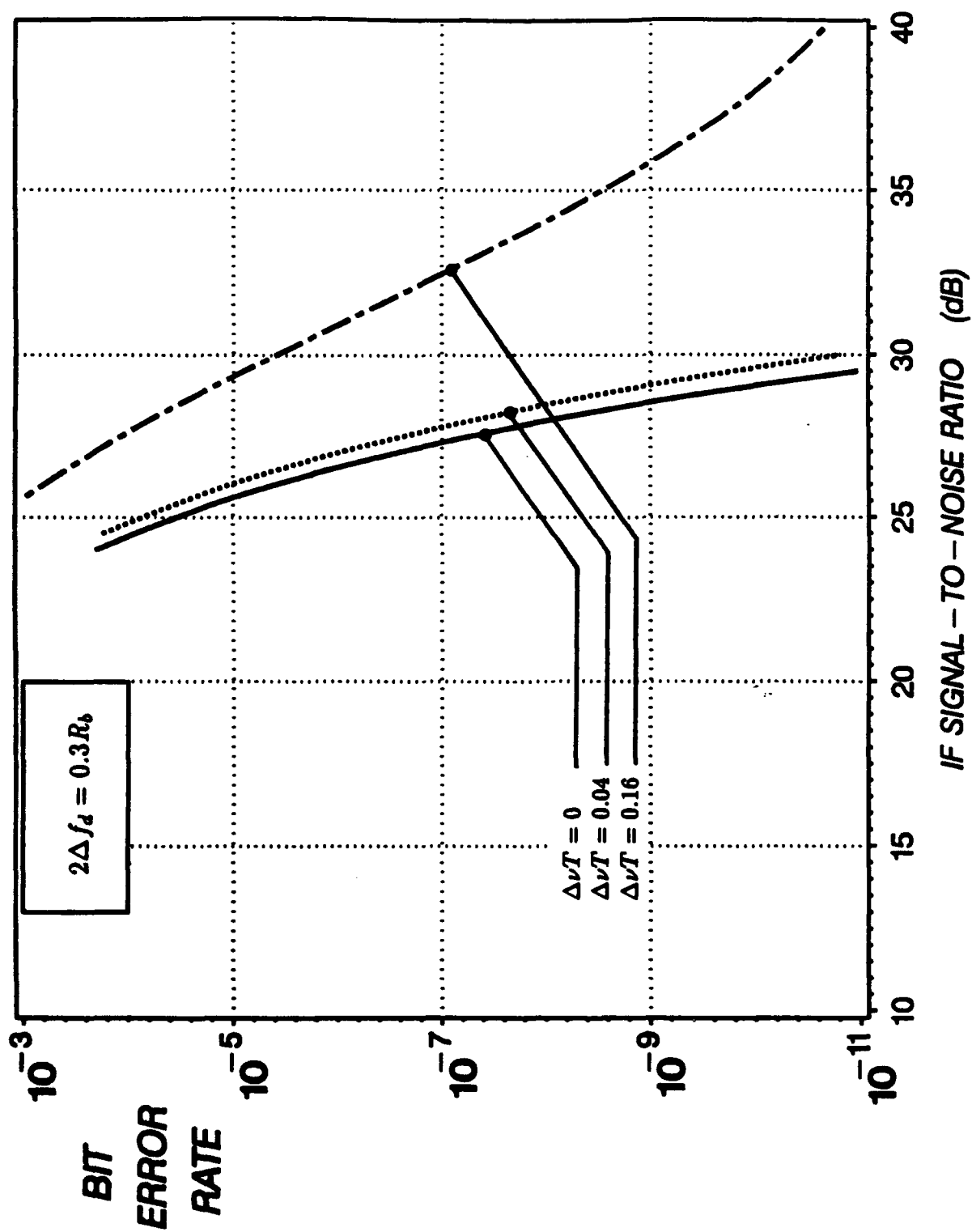


Figure 14



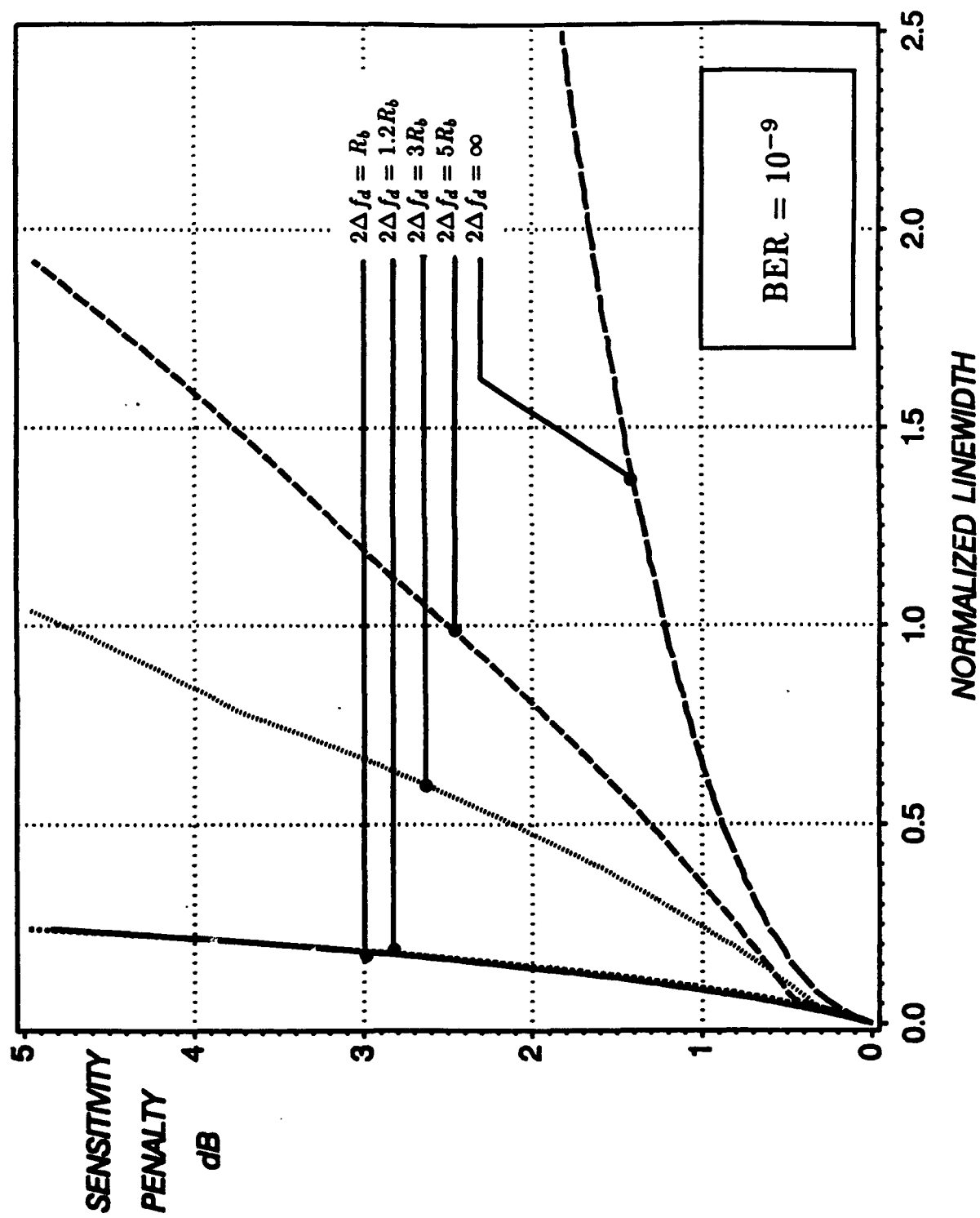


Figure 15

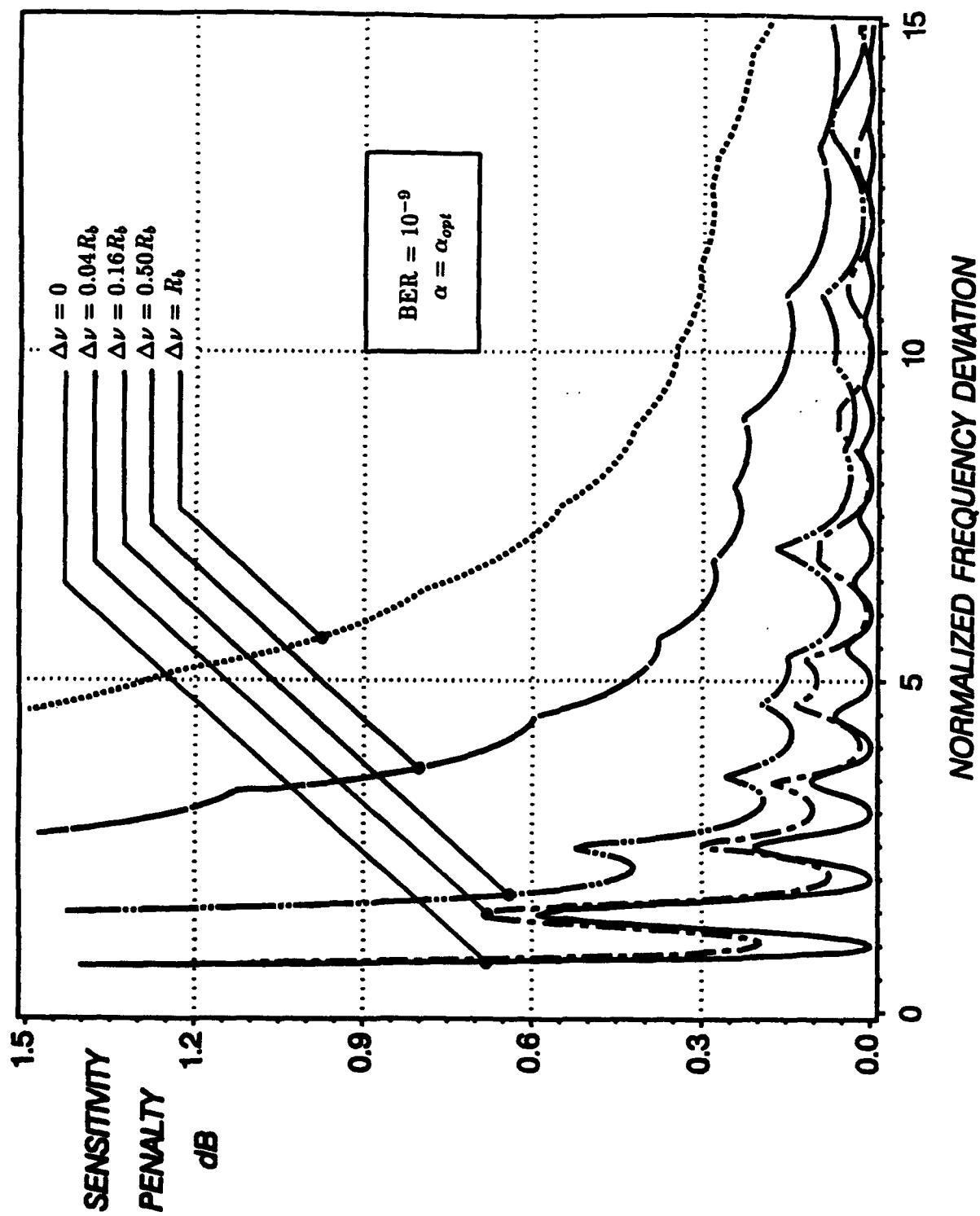


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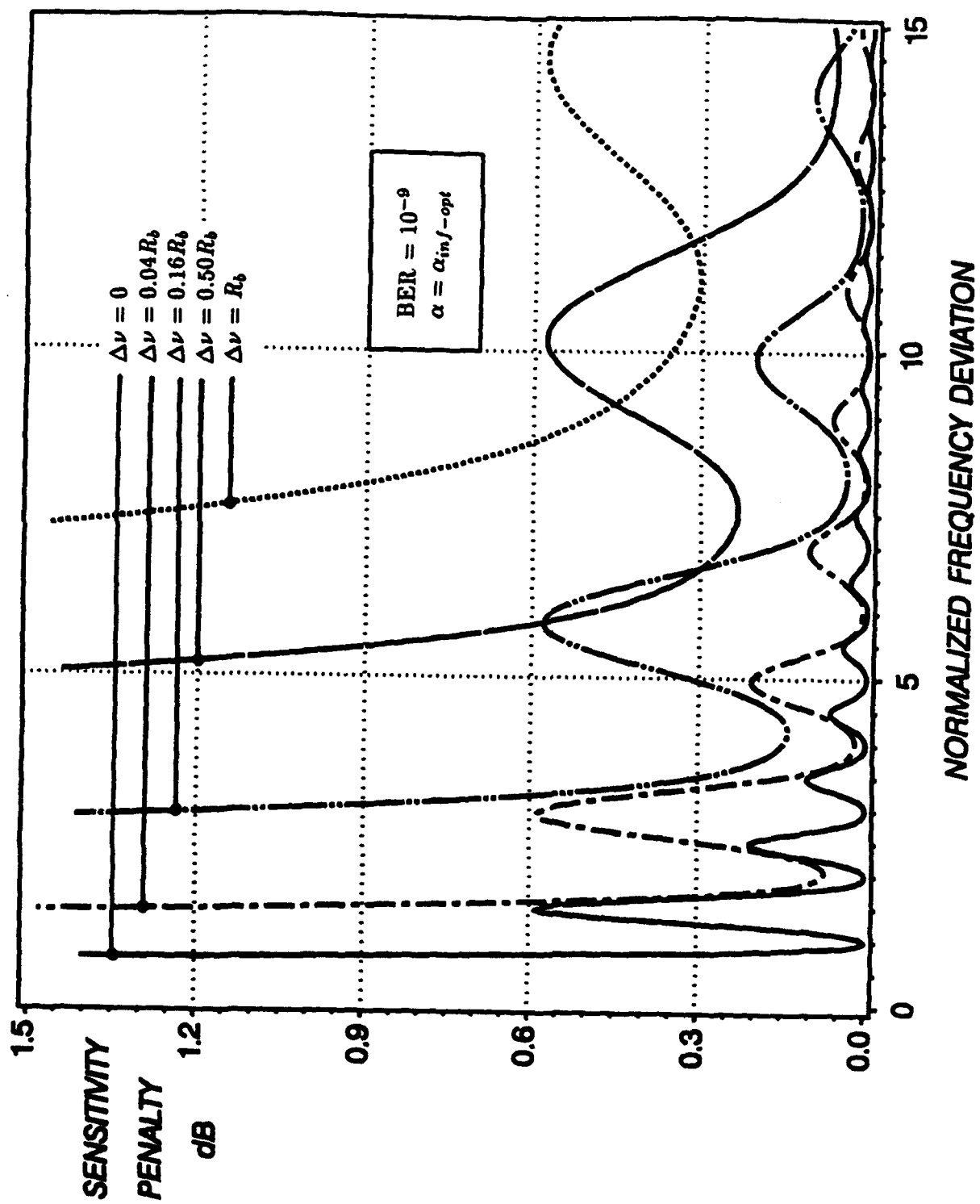


Figure 17

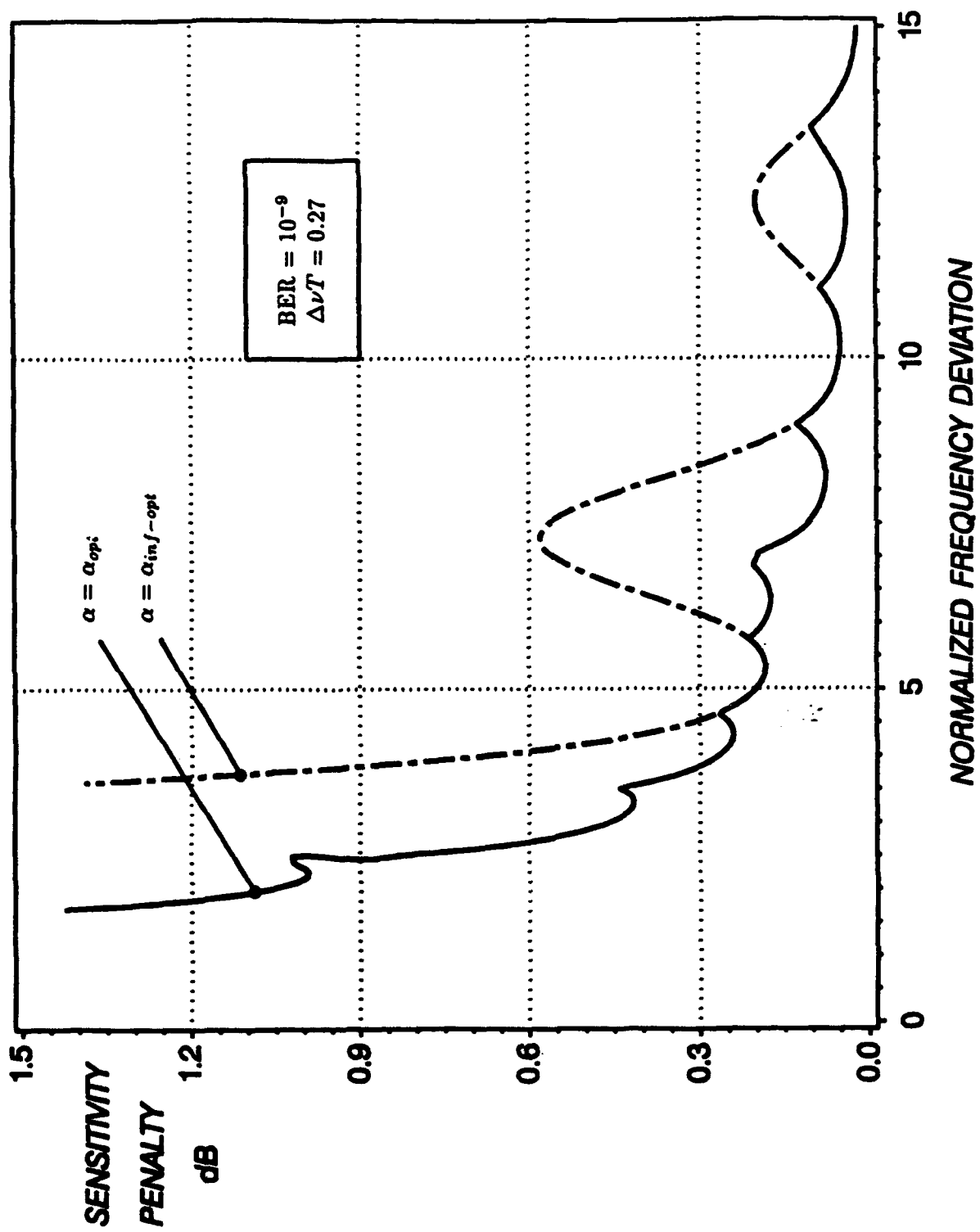


Figure 18